SYSTEMS OF LINEAR EQUATIONS
CHAPTER 3 & 4 DIRECTED NOTES

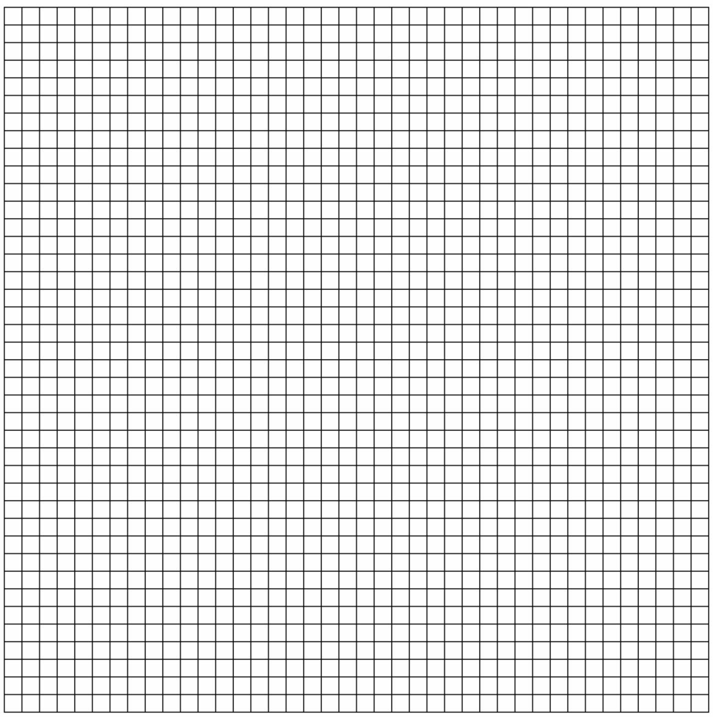
PART A: GRAPHS OF LINEAR SYSTEMS

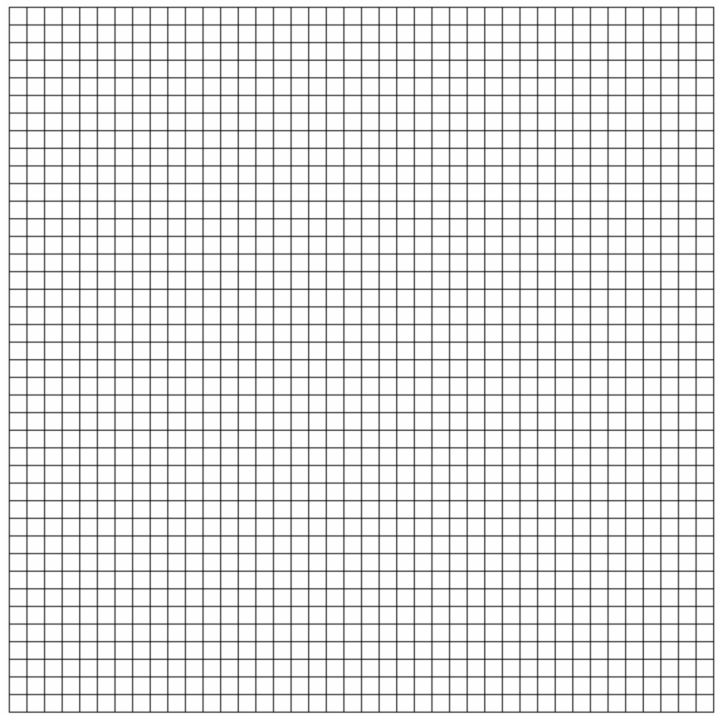
EXAMPLE 1: CONSISTENT & INDEPENDENT SYSTEMS

Graph the following system of equations.

(A) $\begin{cases} 3x + 2y = 14 \\ x - 3y = -10 \end{cases}$

(B) $\begin{cases} 4x - y = -22 \\ 5y = -2x \end{cases}$



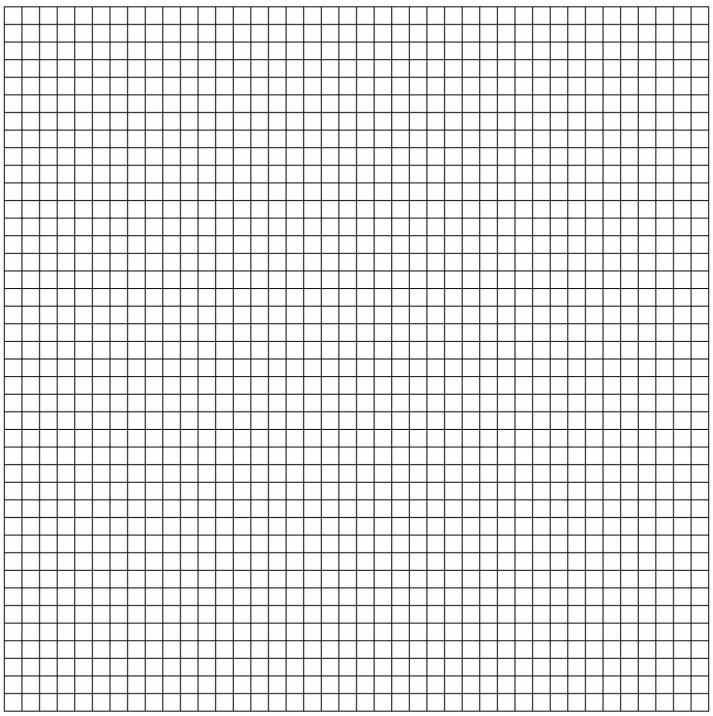


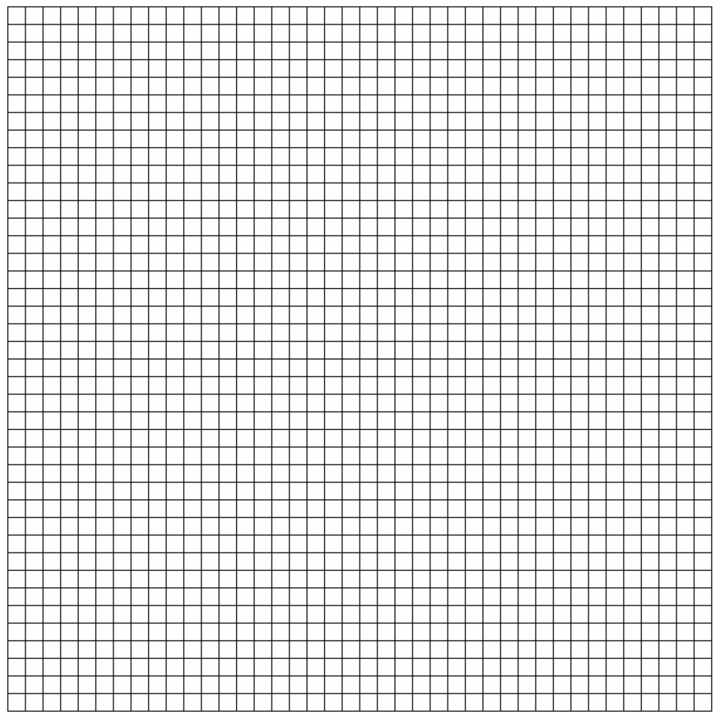
<u>EXAMPLE 2</u>: SPECIAL CASES

Graph the following system of equations.

(A) $\begin{cases} 3x + 2y = 6 \\ 6x + 4y = 8 \end{cases}$

(B) $\begin{cases} 8x + 12y = 20 \\ 6x + 8y = 15 \end{cases}$

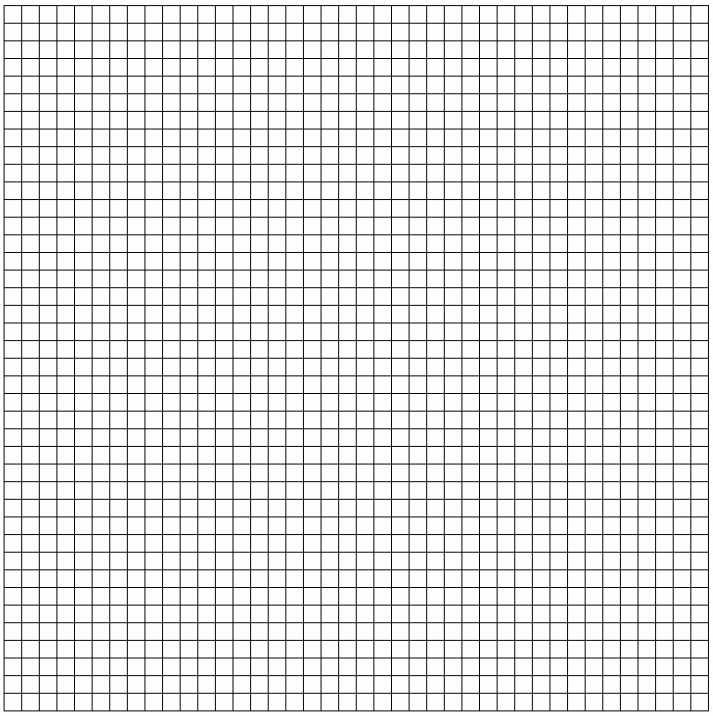




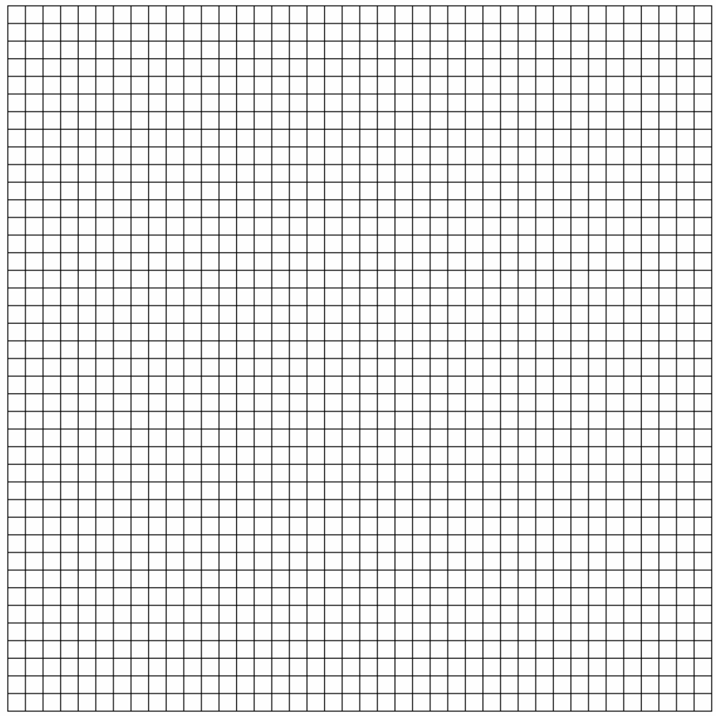
EXAMPLE 3: CONSISTENT & INDEPENDENT SYSTEMS

Graph the following inequalities

(A) $y < 3x + 1$



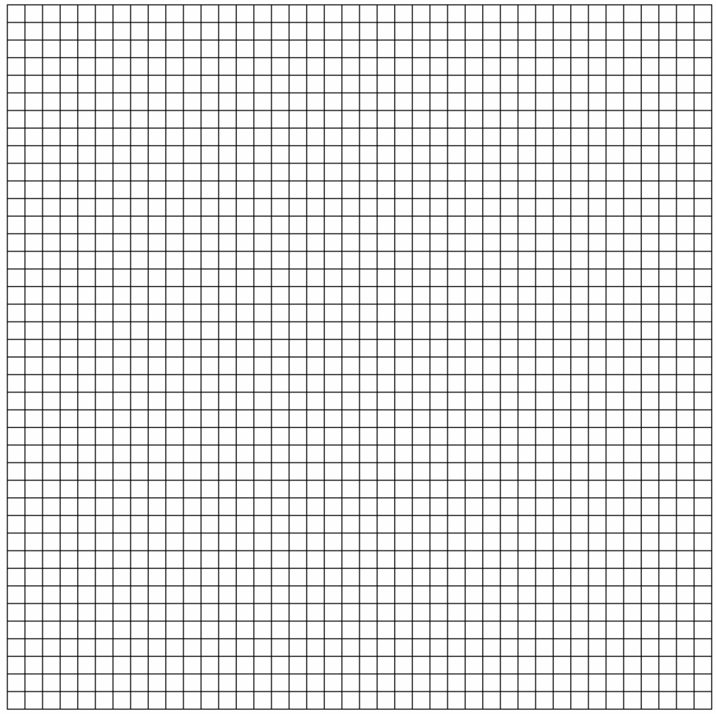
(B) $x - 2y \leq 14$



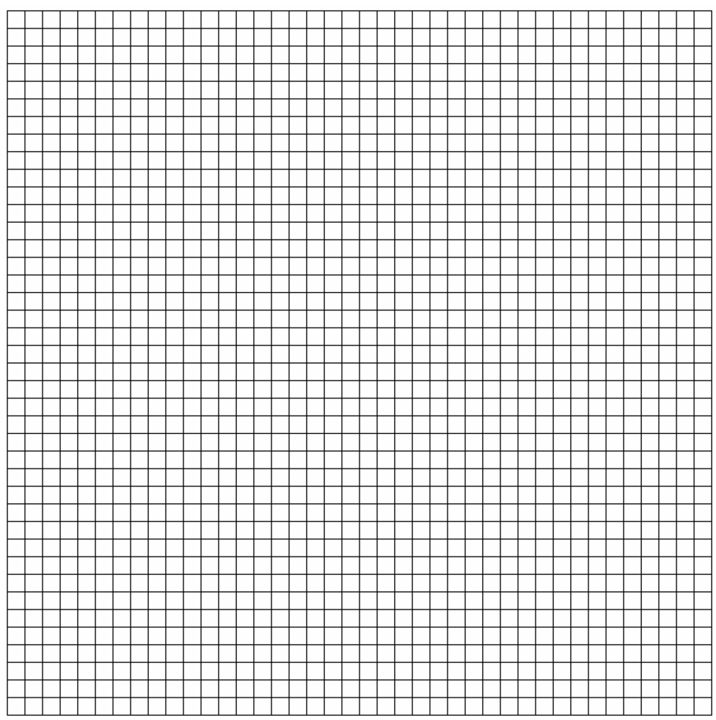
EXAMPLE 4: SYSTEMS OF LINEAR INEQUALITIES

Graph the following system of equations.

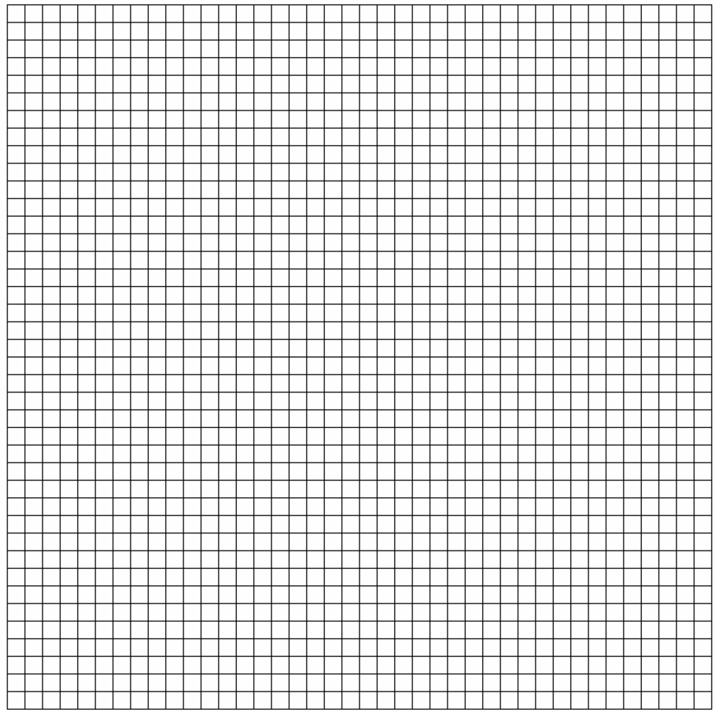
(A) $\begin{cases} x - 4y \geq 8 \\ 5x + 6y > 12 \end{cases}$



(B) $\begin{cases} x \geq 4 \\ y < 8 \\ 2x + 3y \leq 9 \\ 3x - 5y > 10 \end{cases}$



(C) $\begin{cases} y > x \\ 2x + y \le -4 \\ x > 0 \end{cases}$



<u>ASSIGNMENT</u>: p. 161 #13-23 odd
p. 176 #12-27 every 3rd, 29-31, 33-42 every 3rd
pp. 183-184 #9-45 every 3rd

<u>PART B</u>: SOLVING SYSTEMS OF EQUATIONS (2-VARIABLES)

<u>EXAMPLE 1</u>: USING GRAPHING CALCULATOR

Solve the following system of equations using the TI-83 Plus.

$$\begin{cases} 4x - 7y = 18 \\ 6x - 3y = 12 \end{cases}$$

<u>NOTES</u>:

SUBSTITUTION

One method of solving systems of linear equations without the calculator doing all of the work to approximate the answer for you is to use substitution. This procedure is best used when at least one of the equations in your system is solved for one of the variables.

<u>EXAMPLE 2</u>: SOLVING SYSTEMS OF EQUATIONS USING SUBSTITUTION

Solve the following system of equations using substitution.

(A) $\begin{cases} x = 7 \\ 3x + 2y = 15 \end{cases}$

(B) $\begin{cases} y = 2x + 3 \\ 3x + 5y = 9 \end{cases}$

(C) $\begin{cases} y = 2x + 4 \\ y = 4x - 1 \end{cases}$

ELIMINATION

Another method of solving systems of equations is called elimination. This method is usually best used when both equations are in standard form.

<u>EXAMPLE 2</u>: SOLVING SYSTEMS OF EQUATIONS USING ELIMINATION

Solve the following system of equations using elimination.

(A) $\begin{cases} x + 2y = 3 \\ 3x + 5y = 4 \end{cases}$

(B) $\begin{cases} 4x + 2y = 13 \\ 7x + 4y = 23 \end{cases}$

(C) $\begin{cases} 4x + 6y = -1 \\ 8x + 3y = 4 \end{cases}$

<u>EXAMPLE 3</u>: SPECIAL CASES

Solve the following system of equations using any method.

(A) $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = 10 \end{cases}$

(B) $\begin{cases} y = 3x + 5 \\ 6x - 2y = -10 \end{cases}$

<u>ASSIGNMENT</u>: pp. 161-162 #27-39 every 3rd, 41-45 odd
p. 169 #7, 8, 9-33 every 3rd

<u>PART C</u>: MATRIX BASICS WITHOUT USING A CALCULATOR

SECTION 1: BASICS

<u>EXAMPLE 1</u>: DIMENSIONS & MATRIX ADDRESSES

Given the following matrices

$$A=\begin{bmatrix}-6 & 10\\ 7 & 13\end{bmatrix} \qquad B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix} \qquad C=\begin{bmatrix}2 & 4 & 6\\ -5 & 7 & -9\end{bmatrix}$$

$$D=\begin{bmatrix}4 & 16\\ 3 & 12\end{bmatrix} \qquad E=\begin{bmatrix}8 & -5 & 5\\ 6 & 2 & 5\\ 1 & -3 & 4\end{bmatrix} \qquad F=\begin{bmatrix}3 & 1 & 5\\ -10 & -9 & 0\end{bmatrix}$$

Determine the following.

(A) Dimensions of A **(B)** Dimensions of B **(C)** Dimensions of F

(D) a_{21} **(E)** e_{13} **(F)** c_{32}

<u>EXAMPLE 2</u>: BASIC OPERATIONS

Refer to the matrices from example 1 to find the following.

(A) $-3D$ **(B)** $2E$ **(C)** $A+D$

(D) $B+E$ **(E)** $3C+5F$

EXAMPLE 3

Solve the following for *x* and *y*.

(A) $\begin{bmatrix} 3x+1 & 5 \\ 4 & 2y-3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 4 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} 6 & -6 \\ 2x+1 & 9 \end{bmatrix} = \begin{bmatrix} 6 & \frac{3}{5}x \\ -19 & 5y \end{bmatrix}$

EXAMPLE 4: ***This example consists of problems 50-54 on p. 222 of your textbook.***

GEOGRAPHY Tracy and Renaldo both collect maps. Together they have a variety of maps from the 1960s to the 1990s. Matrix M shows the number of each type of map they have.

	'60s	'70s	'80s	'90s
Europe	3	1	4	2
Asia	5	3	6	3
North America	2	7	9	5
Africa	8	5	4	6

$= M$

50. What are the dimensions of matrix M?

51. Describe the entry at m_{42}.

52. Describe the entry at m_{21}.

53. What is the total number of maps of Africa that Renaldo and Tracy have?

54. What is the total number of maps from the 1960s that Tracy and Renaldo have?

ASSIGNMENT: pp. 221-223 #13-29 odd, 30-45 every 3rd, 55-58

SECTION 2: MATRIX MULTIPLICATION, IDENTITY MATRICES & AUGMENTED MATRICES

<u>EXAMPLE 5</u>: DIMENSIONS & MATRIX ADDRESSES

Given the following matrices

$$A=\begin{bmatrix}-6 & 10\\ 7 & 13\end{bmatrix} \qquad B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix} \qquad C=\begin{bmatrix}2 & 4 & 6\\ -5 & 7 & -9\end{bmatrix}$$

$$D=\begin{bmatrix}4 & 16\\ 3 & 12\end{bmatrix} \qquad E=\begin{bmatrix}8 & -5 & 5\\ 6 & 2 & 5\\ 1 & -3 & 4\end{bmatrix} \qquad F=\begin{bmatrix}3 & 1 & 5\\ -10 & -9 & 0\end{bmatrix}$$

Determine the following.

(A) $A \cdot D$ **(B)** $D \cdot A$ **(C)** $B \cdot E$

(D) $E \cdot B$ **(E)** $C \cdot F$ **(F)** $D \cdot F$

IDENTITY MATRICES: Given a matrix *A*. An identity matrix is a **square matrix** such that $A \cdot I_{n\times n} = A$ or $I_{n\times n} \cdot A = A$ (or both).

Examples of Identity Matrices

$$I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_{4\times4} = \qquad I_{5\times5} =$$

<u>EXAMPLE 6</u>: MULTIPLICATION OF AN IDENTITY MATRIX

Given the following matrices

$$A = \begin{bmatrix} -6 & 10 \\ 7 & 13 \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 4 & 6 \\ -5 & 7 & -9 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 16 \\ 3 & 12 \end{bmatrix} \qquad E = \begin{bmatrix} 8 & -5 & 5 \\ 6 & 2 & 5 \\ 1 & -3 & 4 \end{bmatrix} \qquad F = \begin{bmatrix} 3 & 1 & 5 \\ -10 & -9 & 0 \end{bmatrix}$$

Determine the following.

(A) $A \cdot I_{2\times2}$ **(B)** $I_{3\times3} \cdot E$ **(C)** $C \cdot I_{2\times2}$

(D) $C \cdot I_{3\times3}$ **(E)** $I_{2\times2} \cdot C$ **(D)** $I_{3\times3} \cdot C$

(THE REMAINING PART OF SECTION 2 IS OPTIONAL)

EXAMPLE 7: INTRODUCTION TO SOLVING SYSTEMS OF EQUATIONS USING AUGMENTED MATRICES

Solve the following system of equations. $\begin{cases} 3x+4y=-15 \\ 5x-2y=53 \end{cases}$

AUGMENTED MATRIX =

Elementary Row Operations

The following operations produce equivalent matrices, and may be used in any order and as many times as necessary to obtain reduced row-echelon form.

- Interchange two rows.
- Multiply all entries in one row by a nonzero number.
- Add a multiple of one row to another row.

Procedure:

1. The matrix left of the divider is converted to an identity matrix.
2. Convert 1 column at a time.
3. Establish the 1 in the column first then establish the 0's.

EXAMPLE 8:

Solve the following system of equations.

(A) $\begin{cases} x - y = 6 \\ 2x + 7y = 3 \end{cases}$

(B) $\begin{cases} x + y = 6 \\ 2x + 2y = 12 \end{cases}$

(C) $\begin{cases} x + 2y = 1 \\ 2x + 4y = 3 \end{cases}$

(D) $\begin{cases} x + y - z = 4 \\ x - y + 2z = 3 \\ x + z = 4 \end{cases}$

ASSIGNMENT:	pp. 229-231 #7-19 odd, 25 & 26 p. 257 #19-29 odd, 31-35 (OPTIONAL)

<u>PART D</u>: MATRIX OPERATIONS USING A CALCULATOR

SECTION 1: SOLVING SYSTEMS OF EQUATIONS AND BASIC OPERATIONS

<u>EXAMPLE 1</u>: REVIEW

Given the following matrices

$$A=\begin{bmatrix}-6 & 10\\ 7 & 13\end{bmatrix} \qquad B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix} \qquad C=\begin{bmatrix}2 & 4 & 6\\ -5 & 7 & -9\end{bmatrix}$$

$$D=\begin{bmatrix}4 & 16\\ 3 & 12\end{bmatrix} \qquad E=\begin{bmatrix}8 & -5 & 5\\ 6 & 2 & 5\\ 1 & -3 & 4\end{bmatrix} \qquad F=\begin{bmatrix}3 & 1 & 5\\ -10 & -9 & 0\end{bmatrix}$$

Determine the following.

(A) $2E$ (B) $A+D$ (C) $B+E$

(D) $C\cdot E$ (E) $E\cdot C$

INVERSE OF A MATRIX: Given A is a square matrix. A^{-1} is the inverse of matrix A only if the following is true:

$$A\cdot A^{-1}=I_{n\times n}=A^{-1}\cdot A$$

For a matrix to have an inverse the following must be true:

1. The matrix was a square matrix.
2. The determinant of the matrix must be $\neq 0$

EXAMPLE 2: MATRIX INVERSES

Given the following matrices

$$A = \begin{bmatrix} -6 & 10 \\ 7 & 13 \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 4 & 6 \\ -5 & 7 & -9 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 16 \\ 3 & 12 \end{bmatrix} \qquad E = \begin{bmatrix} 8 & -5 & 5 \\ 6 & 2 & 5 \\ 1 & -3 & 4 \end{bmatrix} \qquad F = \begin{bmatrix} 3 & 1 & 5 \\ -10 & -9 & 0 \end{bmatrix}$$

Determine the following.

(A) A^{-1} (B) D^{-1} (C) E^{-1} (D) F^{-1}

EXAMPLE 3: SOLVING SYSTEMS OF EQUATIONS WITH THE CALCULATOR

Solve the following system of equations.

(A) $\begin{cases} 2x + 3y = 7 \\ x - 3y = -1 \end{cases}$

(B) $\begin{cases} 3x + 6y = 15 \\ 2x + 4y = 10 \end{cases}$

(C) $\begin{cases} x + 5y + z = 4 \\ 3x - 5y = 3 \\ 2x + 4y = 0 \end{cases}$

(D) $\begin{cases} x + 3y - 2z = 4 \\ 4x - y + z = -1 \\ 3x - 4y + 3z = -5 \end{cases}$

ASSIGNMENT:	pp. 239-240 #13-23, 36-44 p. 249 #19-28

SECTION 2: POWERS OF MATRICES

FACT: In order to raise a matrix to a power it must be a **square matrix.**

EXAMPLE 4: RAISING MATRICES TO POWERS

Given the following matrices

$A = \begin{bmatrix} -6 & 10 \\ 7 & 13 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 4 & 6 \\ -5 & 7 & -9 \end{bmatrix}$

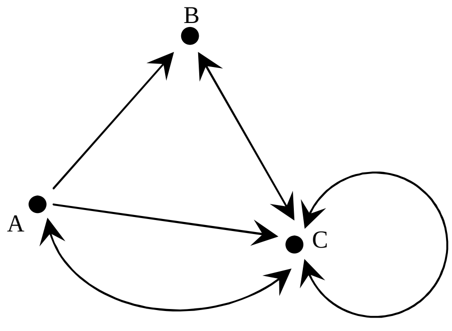
$D = \begin{bmatrix} 4 & 16 \\ 3 & 12 \end{bmatrix}$ $E = \begin{bmatrix} 8 & -5 & 5 \\ 6 & 2 & 5 \\ 1 & -3 & 4 \end{bmatrix}$ $F = \begin{bmatrix} 3 & 1 & 5 \\ -10 & -9 & 0 \end{bmatrix}$

Determine the following.

(A) A^2 (B) B^2 (C) C^3 (D) E^5

EXAMPLE 5: DIRECTED NETWORKS & ADJACENCY MATIRICES

Given the directed network below.



(A) Write the 1-stage adjacency matrix for the directed network.

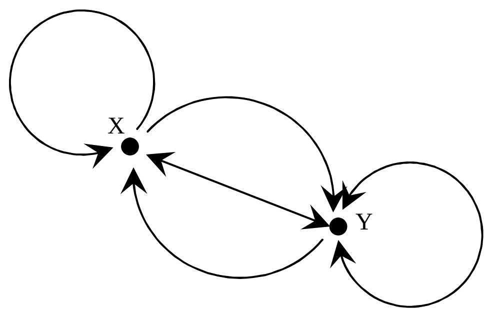
(B) Write the 2-step adjacency matrix for the directed network.

(C) How many 3-stage pathways are from A to C?

ASSIGNMENT: Complete the problems on the following pages.

<u>ASSIGNMENT</u>: Complete the problems below.

For #1-4, refer to the following directed network.



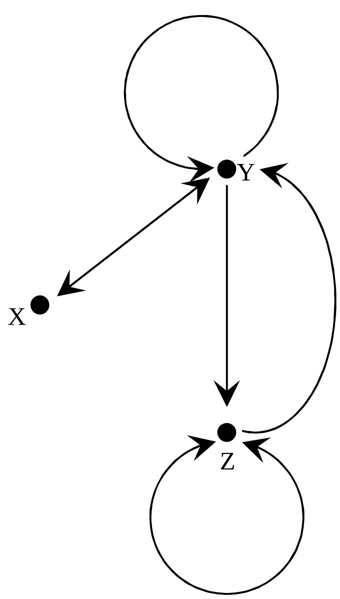
1. Write the 1-stage adjacency matrix for this directed network.

2. How many 1-stage paths start at Y and end at Y?

3. Write the 3-stage adjacency matrix for this directed network.

4. How many 5-stage paths start at X and end at X?

For #5-8, refer to the following directed network.



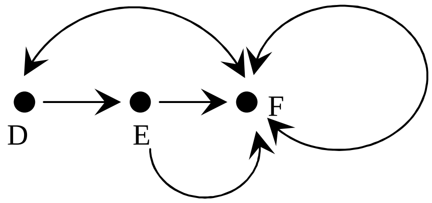
5. How many 2-stage paths start at Y and end at Z?

6. How many 3-stage paths start at X and end at Z?

7. How many 4-stage paths start at X and end at Z?

8. Write the 5-stage adjacency matrix for this directed network.

For #9-10, refer to the following directed network.

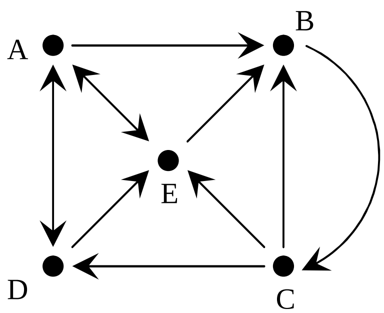


9. Complete the following given that X is the 4-stage adjacency matrix for this directed network.

$$X = \begin{matrix} & D & E & F \\ D & & & \\ E & & & \\ F & & & \end{matrix}$$

10. What does the number with the matrix address x_{23} represent?

For #11-12, refer to the following directed network.



11. How many 4-stage pathways start at D and end at C?

12. How many 6-stage pathways start at A and end at A?

<u>PART E</u>: LINEAR PROGRAMMING

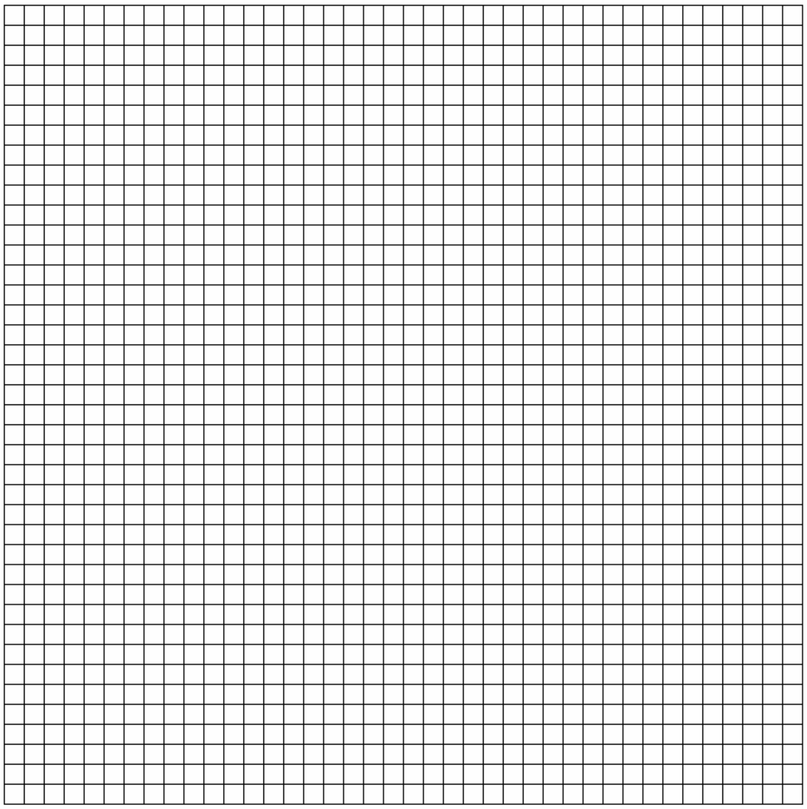
<u>EXAMPLE 1</u>: #33 p. 193 of textbook.

TRANSPORTATION Trenton, Michigan, a small community, is trying to establish a public transportation system of large and small vans. It can spend no more than $100,000 for both sizes of vehicles and no more than $500 per month for maintenance. The community can purchase a small van for $10,000 and maintain it for $100 per month. The large vans cost $20,000 each and can be maintained for $75 per month. Each large van carries a maximum of 15 passengers, and each small van carries a maximum of 7 passengers.

a. Write a system of linear inequalities to represent the constraints.

b. Graph the feasible region.

c. Write the objective function for the number of passengers, and find the maximum number of passengers for the given constraints.



<u>EXAMPLE 2</u>: TYPICALLY-STATED LINEAR PROGRAMMING PROBLEM

This problem is adapted from #35 on page 193 of your textbook.

A school dietician wants to prepare a meal of meat and vegetables that has the lowest possible fat and that meets the Food and Drug Administration recommended daily allowances (RDA) or iron and protein. The RDA minimums are 20 milligrams of iron and 45 grams of protein. Each 3-ounce serving of meat serving of meat contains 45 grams of protein, 10 milligrams of iron, and 4 grams of fat. Each 1-cup serving of vegetables contains 9 grams of protein, 6 milligrams of iron, and 2 grams of fat. So, what should be the serving sizes of meat and vegetables be that satisfy the RDA allowances for protein and iron and has the lowest possible fat?

<u>ASSIGNMENT</u>: Complete the problems on the following pages.

<u>ASSIGNMENT</u>: Solve the following problems. Show all work.

1. A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only $1200 to spend and each acre of wheat costs $200 to plant and each acre of rye costs $100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is $500 per acre of wheat and $300 per acre of rye how many acres of each should be planted to maximize profits?

2. George starts his own small business making USB Flash Drives. It takes 1 hour for George to make a 1 GB Flash Drive and it takes him 1 hour 10 minutes to make a 2 GB Flash Drive. George can only spend 8 hours per day making the Flash Drives. A 1GB Flash Drive requires $2.50 of materials to make while a 2 GB Flash Drive requires $7.50 of materials to make. George wants to keep his operation costs to be no more than $25 per day. George needs to make at least $60 profit per day to keep his business afloat. For each 2 GB Flash Drive he makes he earns $8 profit and for every 1 GB Flash Drive he makes he earns $7.50 profit. How many of each drive should he make in order to earn the highest possible profit?

3. A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least three tons of ore must be processed each day. Ore from source A costs $20 per ton to process, and ore from source B costs $10 per ton to process. Costs must be kept to less than $80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?