

QUADRATIC FUNCTIONS INVESTIGATION

$$y = ax^2 + bx + c$$

PART 1: CHANGING a

1. Open the worksheet titled BASICS in the QUADRATIC FUNCTIONS INVESTIGATION Excel file. [This is the file QUADRATIC FUNCTIONS file found in the EXCEL-WKSHT folder.]

2. Use the scroll bars to set the equation to $y = x^2$. [Set $a = 1$, $b = 0$ and $c = 0$]

Where is the vertex of this parabola?

3. Now use the **red** scroll bar to change the value of a in the equation. How does changing the value of a affect the parabola?

4. Summarize your findings below.

(A) What shape does the parabola have (that is, which way the parabola opens) when $a > 0$? Does the parabola reach a maximum or a minimum value for y at the vertex?

(B) What shape does the parabola have (that is, which way the parabola opens) when $a < 0$? Does the parabola reach a maximum or a minimum value for y at the vertex?

(C) What happens to the graph of the function when $a = 0$? Why does this happen? (Before answering this question, change the values of b and c while $a = 0$)

(D) Does changing the value of a affect the x -coordinate of the vertex?

(E) Does changing the value of a affect the y -coordinate of the vertex?

5. Now use the scroll bars to set the equation to $y = x^2 + 4x + 5$. [Set $a = 1$, $b = 4$ and $c = 5$]

Where is the vertex of this parabola?

6. Now use the **red** scroll bar to change the value of a in the equation.

(A) Does changing the value of a still affect the shape of the parabola?

(B) Does changing the value of a affect the x -coordinate of the vertex?

(C) Does changing the value of a affect the y -coordinate of the vertex?

PART 1 CONCLUSIONS:

Given a quadratic function: $y = ax^2 + bx + c$

- ★ If $a > 0$, the parabola opens _____ and the vertex of the parabola is the _____ value of the function.
- ★ If $a < 0$, the parabola opens _____ and the vertex of the parabola is the _____ value of the function.
- ★ If $a = 0$, the function becomes _____.
- ★ If $b = 0$, the x -coordinate of the vertex is equal to _____ no matter what the value of a is.
- ★ If $b \neq 0$, then changing the value of a will also do what?

PART 2: CHANGING c

1. Open the worksheet titled BASICS in the QUADRATIC FUNCTIONS INVESTIGATION Excel file.
2. Use the scroll bars to reset the equation back to $y = x^2$.
3. Now adjust the **green** scroll bar which changes the c value. How does changing the value of c affect the parabola?
4. Use the scroll bars on the worksheet to set the worksheet to each of the equations listed below in the table. Fill in the missing information.

y					VERTEX	y - INTERCEPT	COORDINATES
a	$x^2 +$	b	$x +$	c			
2	$x^2 +$	4	$x +$	0			(-8,) (2,) (6,)
2	$x^2 +$	4	$x +$	20			(-8,) (2,) (6,)
2	$x^2 +$	4	$x +$	-30			(-8,) (2,) (6,)

5. Use the graphs you saw and table above to answer the following questions.
 - (A) What happens when the equation is changed from $y = 2x^2 + 4x + 0$ to $y = 2x^2 + 4x + 20$?
 - (B) What happens when the equation is changed from $y = 2x^2 + 4x + 0$ to $y = 2x^2 + 4x - 30$?
 - (C) What is the relationship between the value of c and the y -intercept?
 - (D) What happens to the y -coordinate of the vertex by changing c ?
 - (E) What happens to the x -coordinate of the vertex by changing c ?

6. Use the scroll bars on the worksheet to set the worksheet to each of the equations listed below in the table. Fill in the missing information.

y					VERTEX	y - INTERCEPT	COORDINATES
a	$x^2 +$	b	$x +$	c			
-0.4	$x^2 +$	8	$x +$	0			(-8,) (2,) (6,)
-0.4	$x^2 +$	8	$x +$	60			(-8,) (2,) (6,)
-0.4	$x^2 +$	8	$x +$	-85			(-8,) (2,) (6,)

7. Use the graphs you saw and table above to answer the following questions.

- (A) What happens when the equation is changed from $y = -0.4x^2 + 8x + 0$ to $y = -0.4x^2 + 8x + 60$?
- (B) What happens when the equation is changed from $y = -0.4x^2 + 8x + 0$ to $y = -0.4x^2 + 8x - 85$?
- (C) What is the relationship between the value of c and the y -intercept? Is it the same as it was in step #5-C?
- (D) What happens to the y -coordinate of the vertex by changing c ?
- (E) What happens to the x -coordinate of the vertex by changing c ?

PART 2 CONCLUSIONS:

Given a quadratic function: $y = ax^2 + bx + c$

- ★ The c value of the function is the _____ of the parabola.

- ★ If p is added to the value the function $y = ax^2 + bx + c$ and p is positive then the parabola, vertex and all, shifts _____ p units.

- ★ If p is added to the value the function $y = ax^2 + bx + c$ and p is negative then the parabola, vertex and all, shifts _____ p units.
[Remember adding a negative value is the same as subtraction.]

- ★ What effect does c have on the x -coordinate of the vertex of the parabola?

PART 3: CHANGING b

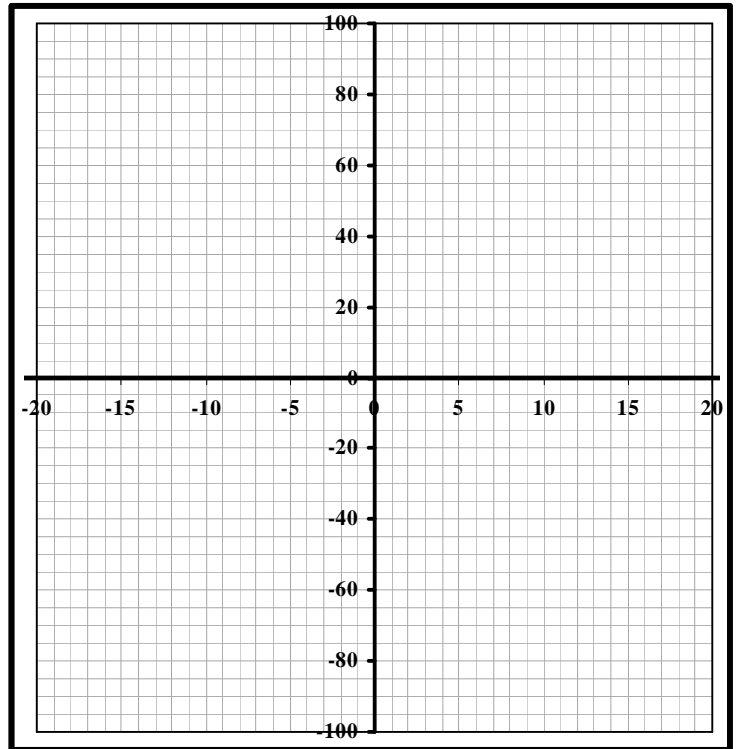
1. Open the worksheet titled BASICS in the QUADRATIC FUNCTIONS INVESTIGATION Excel file.
2. Use the scroll bars to reset the equation to $y = 2x^2 + 0x - 12$ as a starting point.
3. Now use the **blue** scroll bar to change the value of b .

(A) Does changing the value of b affect the shape of the parabola?

(B) As you change the value of b , what do you notice about the movement of the vertex?

4. Use the **blue** scroll bar to change the value of b to those shown in the table below.
Fill in the rest of the information.

EQUATION: $y = 2x^2 + bx - 12$		
b	VERTEX	
	x -coordinate	y -coordinate
-24		
-20		
-14		
-10		
-4		
0		
4		
10		
14		
20		
24		



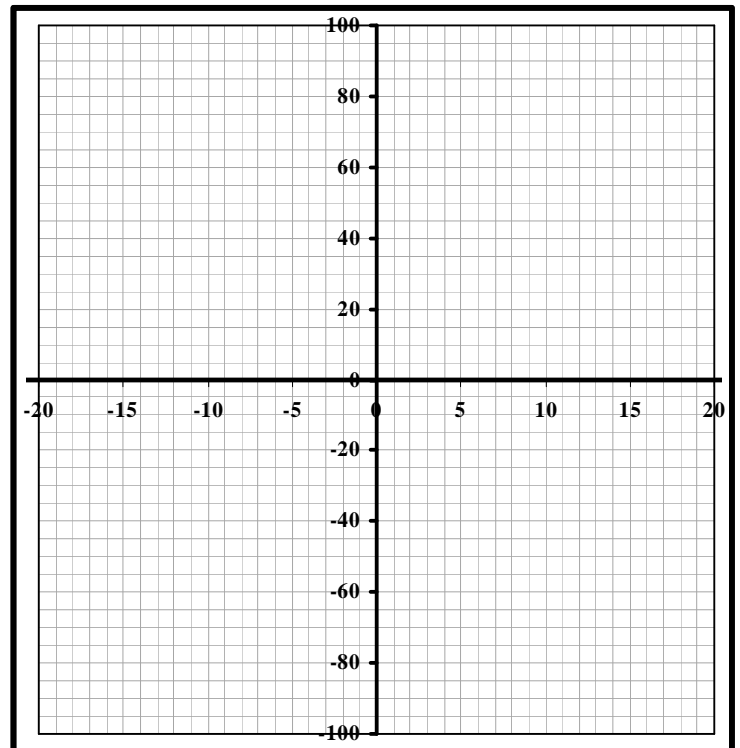
5. Plot the different vertices from the table above on the graph provided. This is the path that the vertex moves along as the value of b changes.

(A) Describe the shape of this path.

(B) Where does the vertex of this path fall?

6. Now set the equation to be $y = -0.3x^2 + 0x + 10$.
7. Now use the **blue** scroll bar to change the value of b . As you change the value of b , what do you notice about the movement of the vertex? Is it different from the last function studied in this section?
8. Use the **blue** scroll bar to change the value of b to those shown in the table below. Fill in the rest of the information.

EQUATION: $y = -0.3x^2 + bx + 10$		
b	VERTEX	
	x -coordinate	y -coordinate
-10.2		
-9		
-6		
-3		
0		
3		
6		
9		
10.2		



9. Plot the different vertices from the table above on the graph provided. This is the path that the vertex moves along as the value of b changes.

(A) Describe the shape of this path.

(B) Where does the vertex of this path fall?

So if a quadratic function in the format $y = ax^2 + bx + c$, changing the b -value causes the vertex to move along path in the shape of a _____ with the vertex located on the _____ axis and coordinates _____.

This means that the path that the vertex of the function moves along as b is changed has its vertex and _____ at the same point.

Let's continue...

10. Open the worksheet titled ADVANCED CHANGE IN B in the QUADRATIC FUNCTIONS INVESTIGATION Excel file.

In this worksheet you will see the following features.

- 2 quadratic function equations, one **red** and one **blue**, each with their own sets of scroll bars that changes their a , b and c values.
- A chart that has three graphs on it (you may not see them all until you start manipulating the red blue functions). The **red** and **blue** graphs correspond to the **red** and **blue** functions that will be manipulated on the page. The **gray** graph corresponds to the path that the vertex of the **blue** function follows as you change the **blue** b values.
- A table with values for x , and three sets of y values, corresponding to the three graphs seen in the chart; **red**, **blue** and **gray**.

11. Set the **blue** function to be $y = 0.5x^2 + 2x + 5$.

12. Notice that the **gray** function changes along with the **blue** function. If you desire you can change the b value on the **blue** function to verify that the vertex moves along the **gray** path as b changes.

13. Now adjust the scroll bars on the **red** function until the **red** graph and the **gray** graph are identical in location and shape, and that the y values for the two functions in the table are also identical.

What is the equation of the path that the vertex of this parabola follows as b is changed? [This is equation of the **red** function once you complete this step.]

14. Now repeat the same steps for the quadratic functions (these will be the **blue** functions you create on the worksheet) listed in the table below and complete the table (using what you found after creating the **red** functions).

EQUATION OF BLUE GRAPH

EQUATION OF RED (& GRAY) GRAPH

y					PATH VERTEX FOLLOWS BY CHANGING b
a	$x^2 +$	b	$x +$	c	
0.5	$x^2 +$	(ANY VALUE)	$x +$	5	$y =$
-3	$x^2 +$	(ANY VALUE)	$x +$	-7	$y =$
1	$x^2 +$	(ANY VALUE)	$x +$	4	$y =$
4.2	$x^2 +$	(ANY VALUE)	$x +$	8	$y =$
-2.1	$x^2 +$	(ANY VALUE)	$x +$	-45	$y =$
-7.8	$x^2 +$	(ANY VALUE)	$x +$	60	$y =$
-0.8	$x^2 +$	(ANY VALUE)	$x +$	0	$y =$
-2	$x^2 +$	(ANY VALUE)	$x +$	6	$y =$
0.1	$x^2 +$	(ANY VALUE)	$x +$	10	$y =$

Now use your findings from the table above to complete the following conclusions.

PART 3 CONCLUSIONS:

Given a quadratic function: $y = ax^2 + bx + c$

Changing the value of b causes the vertex of the parabola to move along a path that has the shape of a _____ with the equation _____.

PART 4: THE VERTEX

In this part of the exploration, we will only be concerned with the x -coordinate of the parabola produced by a quadratic function. The reason is that the y -coordinate is dependent upon the x -coordinate, as well as the values for the constants a , b and c , so any changes to the constants will no doubt change y . The effects on the x -coordinate are a bit more obvious.

Based on your observations made earlier in this exploration...

- ★ Does changing the value of a affect the x -coordinate of the vertex?

- ★ Does changing the value of b affect the x -coordinate of the vertex?

- ★ Does changing the value of c affect the x -coordinate of the vertex?

Therefore, if we are to find a way of defining the x -coordinate of the vertex for a quadratic function, what constant values will be involved?

Now we are going to find out how those values can be used to determine the x -coordinate of the vertex.

Changing a :

1. Open the worksheet titled BASICS in the QUADRATIC FUNCTIONS INVESTIGATION Excel file.
2. Use the scroll bars to adjust the function to have the values noted in the table below. Complete the table below.

y					x - COORDINATE OF VERTEX
a	$x^2 +$	b	$x +$	c	
9	$x^2 +$	-18	$x +$	(ANY VALUE)	
3	$x^2 +$	-18	$x +$	(ANY VALUE)	
1	$x^2 +$	-18	$x +$	(ANY VALUE)	

3. In the table for #2, the value of a was divided by 3 as you move down the table.

How did the x -coordinate of the vertex change as this occurred?

4. Do a similar experiment to complete the table below.

y					x - COORDINATE OF VERTEX
a	$x^2 +$	b	$x +$	c	
1	$x^2 +$	16	$x +$	(ANY VALUE)	
2	$x^2 +$	16	$x +$	(ANY VALUE)	
4	$x^2 +$	16	$x +$	(ANY VALUE)	
8	$x^2 +$	16	$x +$	(ANY VALUE)	

5. In the table for #4, the value of a was multiplied by 2 as you move down the table.

How did the x -coordinate of the vertex change as this occurred?

6. Based on these observations, do a and the x -coordinate of the vertex vary directly or vary inversely?

Changing b :

7. Use the scroll bars to adjust the function to have the values noted in the table below. Complete the table below.

y					x - COORDINATE OF VERTEX
a	$x^2 +$	b	$x +$	c	
-1	$x^2 +$	25	$x +$	(ANY VALUE)	
-1	$x^2 +$	5	$x +$	(ANY VALUE)	
-1	$x^2 +$	1	$x +$	(ANY VALUE)	
-1	$x^2 +$	0.2	$x +$	(ANY VALUE)	

8. In the table for #7, the value of b was divided by 5 as you move down the table.

How did the x -coordinate of the vertex change as this occurred?

9. Do a similar experiment to complete the table below.

y					x - COORDINATE OF VERTEX
a	$x^2 +$	b	$x +$	c	
-2	$x^2 +$	-4	$x +$	(ANY VALUE)	
-2	$x^2 +$	-10	$x +$	(ANY VALUE)	
-2	$x^2 +$	-25	$x +$	(ANY VALUE)	

10. In the table, the value of b was multiplied by 2.5 as you move down the table.

How did the x -coordinate of the vertex change as this occurred?

11. Based on these observations, do b and the x -coordinate of the vertex vary directly or vary inversely?

Interaction of a and b :

12. Use the scroll bars to adjust the function to have the values noted in the table below. Complete the first four rows of the table below.

y					x -COORDINATE OF VERTEX
a	$x^2 +$	b	$x +$	c	
-4	$x^2 +$	-16	$x +$	(ANY VALUE)	
6	$x^2 +$	24	$x +$	(ANY VALUE)	
2.5	$x^2 +$	10	$x +$	(ANY VALUE)	
-0.1	$x^2 +$	-0.4	$x +$	(ANY VALUE)	

13. What do the four functions in the table in #12 have in common?

14. Use the scroll bars to adjust the function to have the values noted in the table below. Complete the first four rows of the table below.

y					x -COORDINATE OF VERTEX
a	$x^2 +$	b	$x +$	c	
10	$x^2 +$	-6	$x +$	(ANY VALUE)	
-2	$x^2 +$	1.2	$x +$	(ANY VALUE)	
-0.5	$x^2 +$	0.3	$x +$	(ANY VALUE)	
2.5	$x^2 +$	-1.5	$x +$	(ANY VALUE)	

15. Do these four functions share the same x -coordinate for the vertex?

16. So clearly, there is a relationship between a and b in determining the x -coordinate of the vertex. Also from parts 1-11 in this part of this exploration, the x -coordinate of the vertex varies inversely with a and varies directly with b . Based on this, write a variation equation between x , a , and b using k as the constant of variation.

17. Notice the tables below are similar to those given to you in steps 12 and 14. Now there are 2 blank rows and an additional column labeled $\frac{b}{a}$. Fill in the information for the first 4 rows of each table.

y					x -COORDINATE OF VERTEX	$\frac{b}{a}$
a	$x^2 +$	b	$x +$	c		
-4	$x^2 +$	-16	$x +$	(ANY VALUE)		
6	$x^2 +$	24	$x +$	(ANY VALUE)		
2.5	$x^2 +$	10	$x +$	(ANY VALUE)		
-0.1	$x^2 +$	-0.4	$x +$	(ANY VALUE)		
	$x^2 +$		$x +$	(ANY VALUE)		
	$x^2 +$		$x +$	(ANY VALUE)		

y					x -COORDINATE OF VERTEX	$\frac{b}{a}$
a	$x^2 +$	b	$x +$	c		
10	$x^2 +$	-6	$x +$	(ANY VALUE)		
-2	$x^2 +$	1.2	$x +$	(ANY VALUE)		
-0.5	$x^2 +$	0.3	$x +$	(ANY VALUE)		
2.5	$x^2 +$	-1.5	$x +$	(ANY VALUE)		
	$x^2 +$		$x +$	(ANY VALUE)		
	$x^2 +$		$x +$	(ANY VALUE)		

18. In each of the two tables, how do the values located in the column “ x -COORDINATE OF VERTEX” and the values in the column $\frac{b}{a}$ relate to one another?

19. Based on your findings in the last step, what is the value of the constant of variation, k , that was referred to in step 16?

20. Based on the observations you made up to this point, to find the x -coordinate of the vertex of a quadratic function you would use the formula:

$$x =$$

21. Now fill in the missing 2 rows of the tables in step 17 and complete the conclusions below. [Use the Excel worksheet to verify that your choices for a and b produce the desired results.]

PART 4 CONCLUSIONS:

Given a quadratic function: $y = ax^2 + bx + c$

★ The x -coordinate of the vertex is given by the formula:

$$x =$$

[Note: Because the vertex also lies on the axis of symmetry, this formula also gives the equation of the axis of symmetry.]

Now take the quiz that is on the QUIZ worksheet of the QUADRATIC FUNCTIONS INVESTIGATION Excel file. Once you complete it, print out a copy to be turned in with this write-up. Make sure you type your name where indicated.

Also, complete the summary page that follows this page.

SUMMARY:

Given a quadratic function with the format $y = ax^2 + bx + c$

- ★ In general, the shape of the function is a _____.
- ★ The y-intercept of the function is given by the value _____.
- ★ Changing the value of a changes the shape of the graph.
 - If $a > 0$, then the parabola opens _____ and y reaches a _____ value.
 - If $a < 0$, then the parabola opens _____ and y reaches a _____ value.
 - If $a = 0$, function is _____.
- ★ Which of the constants have no effect on the shape of the graph?
- ★ Changing the value of b (while leaving a and c unchanged) causes the vertex of the parabola to move along a parabolic path given by the equation: $y =$ _____.
- ★ The x -coordinate, as well as the equation of the axis of symmetry, is given by the formula: $x =$ _____.