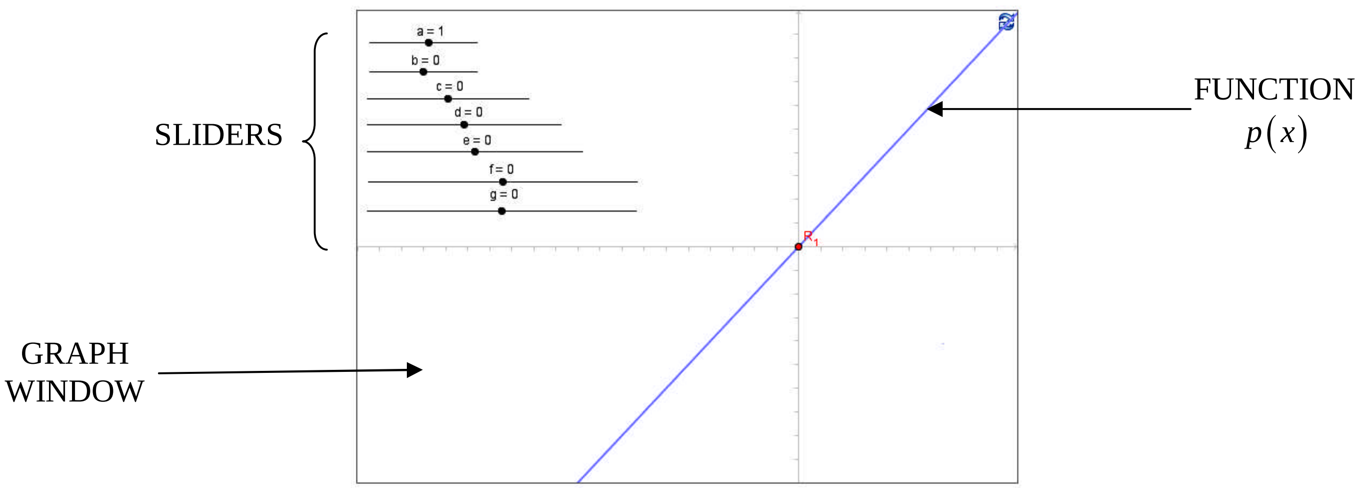
POLYNOMIAL FUNCTION EXPLORATIONS

This exploration uses a Java applet that I created in Geogebra. You do not need to have Geogebra installed on your computer to run this file as it is a stand-alone webpage. The link to the webpage is below:

http://atsorren.freewebpages.org/GENERAL/POLY/poly.html

<u>PART 1</u>: NAVAGATING THE GEOGEBRA APPLET/FILE

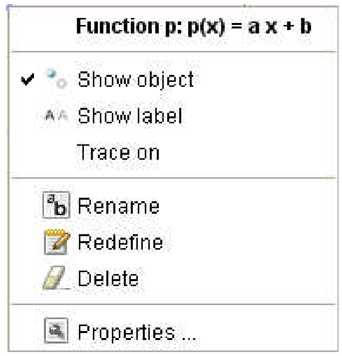
Once you open webpage, you will see a window that looks like the one shown below. **When opening the web page, it may take a few minutes for the page to completely show up and respond as described below, so be patient.**



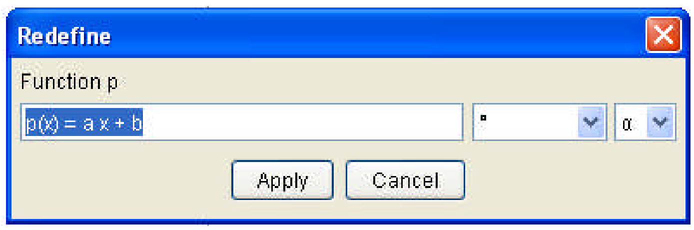
CHANGING THE FUNCTION $p(x)$

When you open this window, the function is set to be $p(x) = ax + b$ so the sliders marked **a** and **b** are the only ones that will have an effect on the function at all. Let's change the function to be $p(x) = (ax+b)(cx+d)$. To do this complete the steps on the following page.

1. Point to the purple line with your mouse (it should become thicker when you do so). Right-click on the line (this is the graph of $p(x)$). A menu box like the one below should appear.



2. Choose "Redefine" from the menu and a dialog box like the one below should appear.



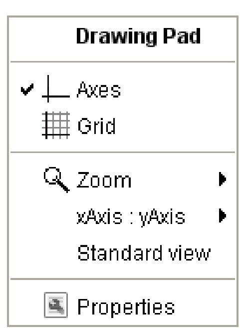
[Do not make any other choice in this menu as any changes you make can alter the applet and make it unable to be used for the purposes of this investigation. If you mistakenly made such a change, then click on the reset icon in the top right corner and the window will be restored to the original settings.]

3. Now type in the desired function: "(a x+b)(c x+d)" **Make sure there is a space between the "a" and the "x", as well as, the "c" and the other "x".** Now you will see the graph change when you move sliders a through c.

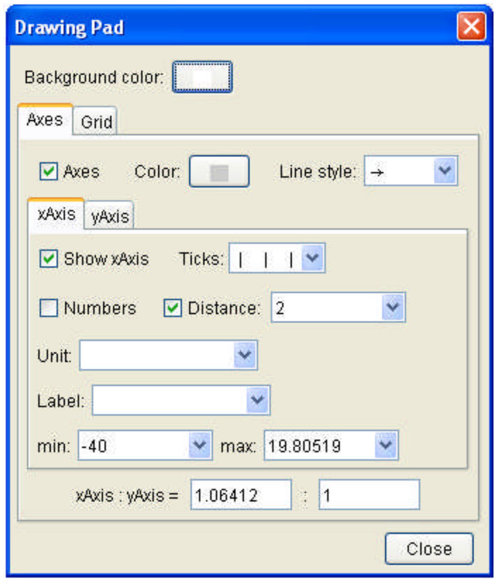
CHANGING THE AXES.

Changing the axis is very similar to how we do this on our calculators. To change the axes, follow the following steps.

1. Right-click anywhere on the GRAPH WINDOW. A menu like the one below should appear.



2. Choose "Properties." A dialog box like the one below will appear.



3. With the "xAxis" tab selected, set "min:" to -40 and set "max:" to 20. Set "Distance:" to 2. [You may have to click on the checkbox to activate this control.]

4. Click the "yAxis" tab. Change the settings [if needed] to the following.

   min: -100
   max: 100
   Distance: 10

5. Click on "Close" once you are finished changing the settings.

Feel free at this point to experiment by redefining $p(x)$ to be other functions, moving the sliders, and changing the axes.

<u>PART 2</u>: DEFINITIONS AND TERMS

We will be looking for certain features of the graphs. To know what we are looking for, we will have to know what these terms and features are.

POLYNOMIAL FUNCTION: A function expressed with the format:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

[We will be using $p(x)$ for our function, but the letter; whether it be *p*, *f*, or anything else; is arbitrary.]

DEGREE OF A POLYNOMIAL: The highest exponent value of a polynomial function.

Example: $f(x) = 3x^5 - 4x^3 + 6x^2 - 4x + 1$ is a degree 5 polynomial function

ROOT: The root of a polynomial is a value of x that makes the value of $f(x) = 0$. Because of this, roots are found on a graph by looking anywhere that the graph crosses the x-axis.

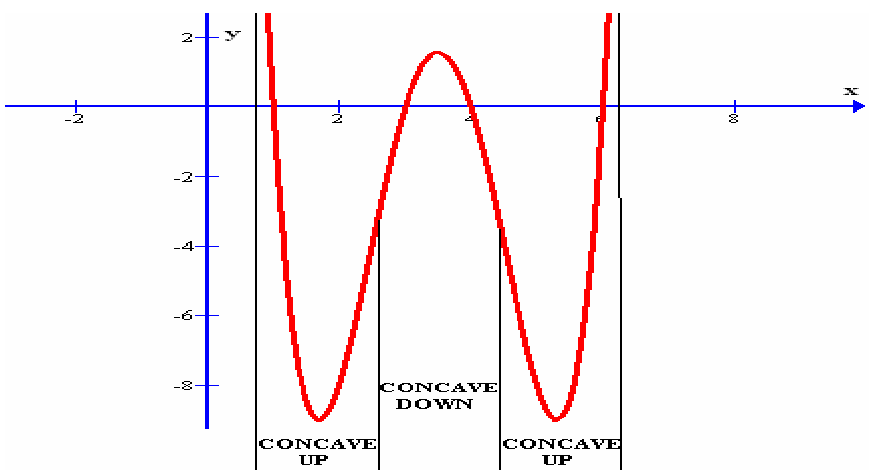
EXTREMUM: This is a point on a graph where the graph changes direction (from increasing to decreasing and vice versa). The two types of extremum are MINIMA (plural of Minimum) and MAXIMA (plural of Maximum).

MINIMUM: A point on the graph where the function changes from decreasing to increasing.

MAXIMUM: A point on the graph where the function changes from increasing to decreasing.

[We always determine if a function is increasing or decreasing by tracing the graph from left to right.]

CONCAVITY: This term describes the direction a polynomial graph is opening. See the picture below.



INFLECTION POINT: A point on the graph where the graph changes from being concave up to concave down (or vice versa).

As we progress through using the applet, be aware of the following markings.

R_1, R_2, R_3,... will be used to mark the locations of real roots.

E_1, E_2, E_3,... will be used to mark the locations of extremum.

I_1, I_2, I_3,... will be used to mark the locations of inflection points.

POLYNOMIAL TYPES:

LINEAR	degree 1
QUADRATIC	degree 2
CUBIC	degree 3
QUARTIC	degree 4

Most texts simply refer to the type of polynomial by its degree for polynomials that have degrees over 4.

<u>PART 3</u>: POLYNOMIAL GRAPH BEHAVIOR

Now use the Web page/Geogebra file to explore polynomials of degree 1-6. You will complete the table on the next page with your findings. To do this, follow the following steps.

1. Enter in the function as defined in the table.
2. Use the scroll bars to adjust the settings of the variables in use. Try to find the functions that have the <u>maximum</u> number of roots, extremum, and inflection points.
3. To assist you with this, use the graph windows as defined below so you can see the various points.

x-Axis settings (for all functions): Minimum: -40 Maximum: 20 Distance: 2

y-Axis settings:

Degree 1	Minimum: -20	Maximum: 20	Distance: 2
Degree 2	Minimum: -100	Maximum: 100	Distance: 10
Degree 3	Minimum: -1000	Maximum: 1000	Distance: 100
Degree 4	Minimum: -10000	Maximum: 10000	Distance: 1000
Degree 5	Minimum: -100000	Maximum: 100000	Distance: 5000
Degree 6	Minimum: -100000	Maximum: 100000	Distance: 5000

DEGREE	FORMAT	MAXIMUM # REAL ROOTS	MAXIMUM # EXTREMUM	MAXIMUM # INFLECTION POINTS	BEGINS AND ENDS [SAME DIRECTION/DIFFERENT DIRECTIONS]
1	$p(x) = ax + b$				
2	$p(x) = ax^2 + bx + c$				
3	$p(x) = ax^3 + bx^2 + cx + d$				
4	$p(x) = ax^4 + bx^3 + cx^2 + dx + e$				
5	$p(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$				
6	$p(x) = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$				
7	$p(x) = a_7x^7 + a_6x^6 + \cdots + a_2x^2 + a_1x + a_0$				
8	$p(x) = a_8x^8 + a_7x^7 + \cdots + a_2x^2 + a_1x + a_0$				
9	$p(x) = a_9x^9 + a_8x^8 + \cdots + a_2x^2 + a_1x + a_0$				
10	$p(x) = a_{10}x^{10} + a_9x^9 + \cdots + a_2x^2 + a_1x + a_0$				
*****	*****	*****	*****	*****	*****
n	$p(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$				

[To fill in the last 5 rows, use patterns that you find for degrees 1-6]

SUMMARY:

Given a polynomial function of degree *n*, then the following is true.

1. The polynomial can have at most __________________ roots.

2. The polynomial can have at most __________________ extremum.

3. The polynomial can have at most __________________ inflection points.

4. If the degree of the polynomial is even, then the function will begin and end in (Circle One) the same direction / different directions.

5. If the degree of the polynomial is odd, then the function will begin and end in (Circle One) the same direction / different directions.

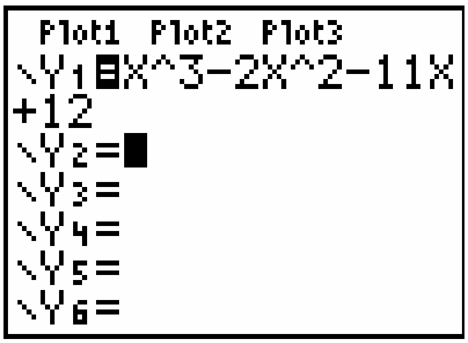
You no longer need the applet, the rest of this exploration will be completed using your graphing calculator.

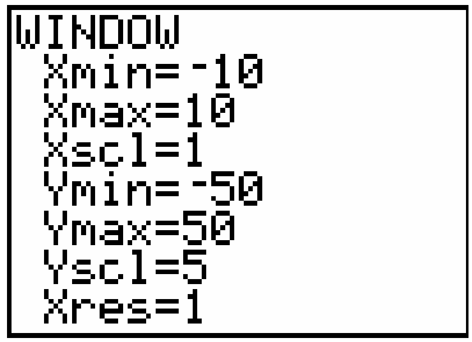
PART 4: FINDING REAL ROOTS AND EXTREMUM

Finding the roots and extremum is quite easy using the TI-83 Plus. The calculator is not set up to find inflection points, however, that is a topic and is more of a concern when taking Calculus. So we will learn how to find the roots and extremum graphically using the TI-83 Plus.

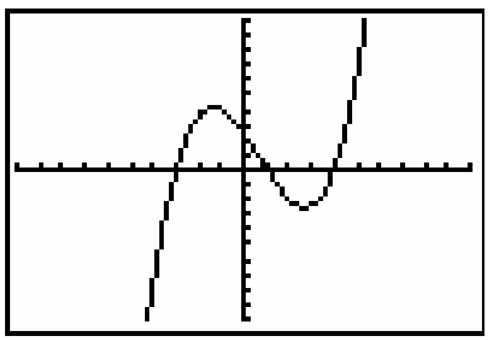
Before doing this set up your calculator as shown below to explore the function

$$f\left(x\right)=x^3-2x^2-11x+12$$





The graph of the function should look like the screen shot shown below.



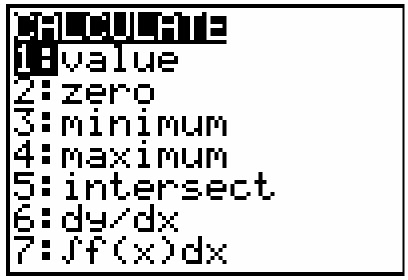
As you can see this function has 3 real roots (as it crosses the *x*-axis 3 times), has one maximum and one minimum. Based on what we learned in Part 3, we do not expect to find any more roots or extremum on the graph, so this is a suitable window for our exploration. Any time you do this on your own, you will need to find a window that allows you to see all of the necessary features of the polynomial.

FINDING REAL ROOTS

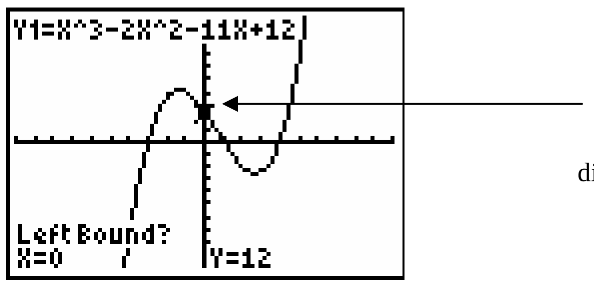
Many students mistakenly use the TRACE feature on the calculator to do this. However, the cursor used in that feature is not guaranteed to touch any of the roots. Therefore, we need to perform operations that give us the exact values of the *x*-intercepts.

Let’s find the first root of this function (the first instance of the graph crossing the *x*-axis as we move from left to right).

1. With the graph of the function still on the calculator viewing window press [2nd] and then [TRACE] to bring up the CALC menu. Your screen should look as shown below.

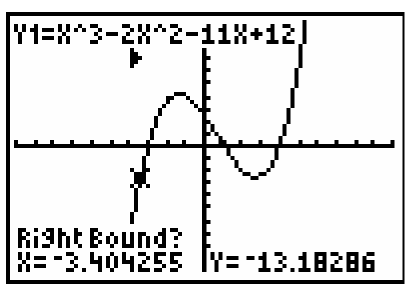


2. Choose **2:zero** since we are looking for a zero value of the polynomial (which is what a root is). Your screen should now look something like this.



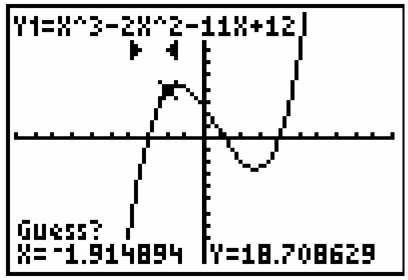
Blinking Cursor
(this may be in a different location on your calculator.)

3. The calculator asks for a “Left Bound.” So you want to move the cursor (using the arrow keys) along your graph until the cursor is anywhere to the left of the root. Press [ENTER]. You can also type in a number that you know is definitely to the left of the root. In either case, the screen should now look something like this.

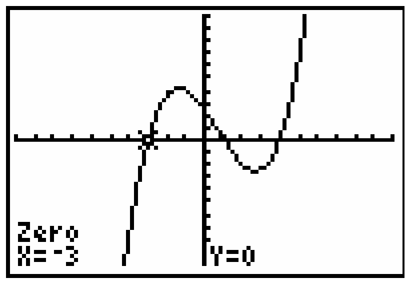


4. An arrow marks your Left Bound and then the calculator asks for a “Right Bound.” Similar to the left bound, now you are to move the cursor (or choose an appropriate value) so it is anywhere to the right of the root (do not go too far as you only want 1 root between your bounds.) Press [ENTER].

Now your view screen looks something like this.



5. Now the calculator asks for a "Guess." This means to move the cursor as close to the root as you can get it. Technically though, as long as you have the cursor anywhere between your Left Bound and Right Bound arrows, this process will work. Press [ENTER].



The coordinates of the root (or zero) are given.

NOTE: Because of the way that the calculator actually determines the value some slight deviations may be seen. Therefore:

(A) If you get a Y value that looks like "1.23E-12" (which indicates scientific notation) note that this and numbers like it can be taken to be 0.

(B) Sometimes the X values will be represented as decimals when they should be exact. For instance, a root of 3 may be shown as "2.999999" or "3.0000001." Therefore, if you get decimals like that, it is safe to round appropriately.

FINDING EXTREMUM

Finding extremum follows a similar process as finding the real roots did. The only exception is that you will choose either **3:minimum** or **4:maximum** from the CALC menu on the TI-83 Plus. The procedures for these are performed the same way as zeros.

EXERCISES

Find all real roots and the coordinates of the extremum for the following functions [if possible].

1. $f(x) = x^3 - 6x^2 - x + 30$

2. $f(x) = 12x^4 + 4x^3 - 49x^2 - 26x + 24$

3. $f(x) = -4x^2 + 5x + 4$

4. $f(x) = 20x^5 - 36x^4 - 71x^3 + 51x^2 - 261x + 297$

5. $f(x) = 9x^6 - 27x^5 - 13x^4 + 68x^3 - 92x^2 + 160x + 80$

6. $f(x) = 3x^4 - 6x^3 - 39x^2 + 42x + 102$

7. $f(x) = 5x^6 - 40x^4 - 80x^2 + 640$

8. $f(x) = x^3 + 1$

9. $f(x) = x^7 + 4x^6 + 5x^4 + 20x^3 - 6x - 24$

10. $f(x) = 5x^5 + 7x^3 - 4x$