

GRAPH TRANSFORMATIONS

In mathematics, we always look for short-cuts to produce graphs. One such shortcut is to make use of graph transformation techniques. This allows us to start with a simple known function (called the **parent function**) and make a few defined changes to it to quickly come up with a graph of a related function (something that I usually call an **offspring function**). We will be exploring these techniques in this investigation

In order to complete this investigation, you will need to access the Excel file entitled GRAPH TRANSFORMATIONS. This is the file GRAPH TRANSFORMATIONS found in the EXCEL-WKSHT folder. [See the DOWNLOADING EXCEL-WKSHT document for instructions in obtaining this file.]

GENERAL FUNCTIONS: $y = a \cdot f(bx + c) + d$

Open the tab labeled “y = f(x).” Even though you can only see one graph at first there are two there. The red function is the parent function $y = f(x)$ [which is the same as $y = 1 \cdot f(1 \cdot x + 0) + 0$]. The “f()” is the grouping symbol which defines the general shape of the function. The grouping symbol defines the shape of the function in ALL functions.

The second function is dark blue. It is set, upon opening the file, to be the same as the parent function. You will be manipulating this function using the scroll bars to see how the changes you make change the shape and location of the function.

PART 1: SHIFTS

(1) Changing d :

- (A) Use the scroll bar to change the value of d . What happens to the graph as you change d ?
- (B) When d is increased, which way does the graph move?
- (C) When d is decreased, which way does the graph move?
- (D) Make the value of $d = 3$. Comparing the graph to the parent function...
 - (i) How has the graph shifted?
 - (ii) How have all of the y-coordinates changed?
- (E) Make the value of $d = -4$. Comparing the graph to the parent function...
 - (i) How has the graph shifted?
 - (ii) How have all of the y-coordinates changed?

Now change the function back to “y = f(x).”

Changing c :

- (A) Use the scroll bar to change the value of c . What happens to the graph as you change c ?
- (B) When c is increased, which way does the graph move?
- (C) When c is decreased, which way does the graph move?

- (D) Make the value of $c = 3$. Comparing the graph to the parent function...
 - (i) How has the graph shifted?
 - (ii) How have all of the x -coordinates changed?

- (E) Make the value of $c = -4$. Comparing the graph to the parent function...
 - (i) How has the graph shifted?
 - (ii) How have all of the x -coordinates changed?

SUMMARY:

1. Increasing the value of d causes the function to shift in what direction?
2. Increasing the value of d changes which coordinate? How?
3. Decreasing the value of d causes the function to shift in what direction?
4. Decreasing the value of d changes which coordinate? How?
5. Increasing the value of c causes the function to shift in what direction?
6. Increasing the value of c changes which coordinate? How?
7. Decreasing the value of c causes the function to shift in what direction?
8. Decreasing the value of c changes which coordinate? How?

FURTHER NOTES:

- ★ Notice that the VERTICAL shifts (which affect the y -coordinates) are defined by d which is **outside** the grouping symbol.
- ★ Notice that the HORIZONTAL shifts (which affect the x -coordinates) are defined by c which is **inside** the grouping symbol.
- ★ Furthermore, notice that changing c (which is inside the grouping symbol) has the ***opposite effect*** on the x -coordinates than what may have been expected [i.e. increasing c caused the x -coordinates to decrease and vice versa]. Changing d , on the other hand, behaves as expected [i.e. increasing d causes the y -coordinates to increase and vice versa].

PART 2: STRETCHES/COMPRESSIONS

Return the function back to “ $y = f(x)$.”

(1) Changing a :

As you make changes described below, keep the values of a **positive**. The effects of making a negative will be discussed later.

(A) Use the scroll bar to change the value of a . What happens to the graph as you change a ? Pay attention to the fact that the y -coordinates are the only ones changing.

(i) When a is increased, how is the graph affected?

(ii) When a is decreased, how is the graph affected?

(B) Make the value of $a = 2$. Comparing the graph to the parent function...

(i) How has the graph changed?

(ii) How have all of the y -coordinates changed?

(C) Make the value of $a = 0.5$. Comparing the graph to the parent function...

(i) How has the graph changed?

(ii) How have all of the y -coordinates changed?

Return the function back to “ $y = f(x)$.”

(2) Changing b :

As you make changes described below, keep the values of b **positive**. The effects of making b negative will be discussed later.

(B) Use the scroll bar to change the value of b . What happens to the graph as you change b ? Pay attention to the fact that the x -coordinates are the only ones changing.

(i) When b is increased, how is the graph affected?

(ii) When b is decreased, how is the graph affected?

(B) Make the value of $b = 2$. Comparing the graph to the parent function...

(i) How has the graph changed?

(ii) How have all of the x -coordinates changed?

(C) Make the value of $b = 0.5$. Comparing the graph to the parent function...

(i) How has the graph changed?

(ii) How have all of the x -coordinates changed?

SUMMARY:

1. Increasing the value of a causes the function to **STRETCH** in what direction?
2. Decreasing the value of a causes the function to **COMPRESS** in what direction?
3. Changing the value of a requires the y -coordinates of the parent function to be (choose one) multiplied by a /divided by a .
4. Increasing the value of b causes the function to **COMPRESS** in what direction?
5. Decreasing the value of b causes the function to **STRETCH** in what direction?
6. Changing the value of b requires the x -coordinates of the parent function to be (choose one) multiplied by b /divided by b .
7. Does the “opposite effect” seen in the horizontal shifts in Part 1 hold true for the horizontal stretches and compressions seen in Part 2?

FURTHER NOTES:

- ★ Sometimes it is difficult to tell the difference between a VERTICAL STRETCH and a HORIZONTAL COMPRESSION (and vice versa) in most functions. This is due to the interaction of the coefficients and the order of operations.

PART 3: $a = 0$ or $b = 0$

What happens to the graph when $a = 0$ or $b = 0$? Why do you think this occurs?

PART 4: REFLECTIONS

Return the function back to “ $y = f(x)$.”

- (1) What happens to the graph when a changes from positive to negative?

- (2) What happens to the graph when b changes from positive to negative?

These are called REFLECTIONS. The reason is the same as in Part 2, you are either multiplying or dividing the coordinate (x or y), but this time the multiplier is negative.

SUMMARY:

1. Changing the value of a to a negative number causes a (choose one) VERTICAL/HORIZONTAL reflection.

2. Changing the value of b to a negative number causes a (choose one) VERTICAL/HORIZONTAL reflection.

PART 5: MULTIPLE TRANSFORMATIONS

(1) VERTICAL TRANSFORMATIONS

(A) Reset the function to be equal to the parent function.
[$a = 1$, $b = 1$, $c = 0$, and $d = 0$]

(B) Let the value of $a = 3$ and $d = -2$. What should happen to the y -coordinates due to these changes?

(C) Did the vertical transformations obey the order of operations?

(2) HORIZONTAL TRANSFORMATIONS

(A) Starting with the function you have from the previous steps, let $b = 2$ and let $c = 4$. What should happen to the x -coordinates due to these changes?

(B) Did the horizontal transformations obey the order of operations?

IMPORTANT FACT: All graph transformations from operations inside the grouping symbols (namely, horizontal transformations) obey the "opposite effect." That means the order of operations is also reversed.

(C) Using the fact above, does reversing the order of operations along with what you noted in (A) account for what you have seen in the changes to the x -coordinates?

FURTHER NOTES:

- ★ When performing graph transformations it is best to perform the vertical transformations and horizontal transformations in separate steps. It does not matter in which order they are done, just as long as they are done separately.
- ★ Graph transformations are defined by additions, subtractions, multiplications, and divisions ONLY. Any other symbols (like exponents and, as we will see later in this course, logarithms and trigonometric functions) are simply part of the grouping symbol. That means they only determine the general shape of the function.

EXERCISES:

Given the table that defines the parent function $y = g(x)$ below:

$$y = g(x)$$

x	y
-7	4
-5	3
-2	-8
2	-6
4	-4
8	2
11	5

Use what you have learned in Parts 1-5 to perform the necessary calculations to complete the tables for the given functions below.

1. $y = 2g(x+1)$

x	y

2. $y = g(x-1) + 4$

x	y

3. $y = -g(2x) - 3$

x	y

4. $y = -2g\left(\frac{1}{2}x - 3\right) + 4$

x	y

PART 6: FURTHER EXPLORATIONS

Explore the other 2 functions in the Excel file. Here the x -coordinates will remain static in the tables (they will not change). The reasons we do this is because, unlike the first function, $y = x^2$ and $y = |x|$ have infinite domains (they are defined for ALL possible values of x .) So even though the transformations will cause the same effect on the x -coordinates, other x -coordinates can and will change to become the old value of another. As a result, be careful when answering some of the questions regarding x -coordinates.

1. Choose the tab $y = x^2$.

(A) Set $a = 2$. What was the effect?

- (i) The graph stretched vertically by a factor of 2 because the y -coordinates were doubled.
- (ii) The graph stretched horizontally by a factor of 2 because the x -coordinates were doubled.
- (iii) The graph compressed vertically by a factor of 2 because the y -coordinates were cut in half.
- (iv) The graph compressed horizontally by a factor of 2 because the x -coordinates were cut in half.

(B) Set $a = 1$ and $b = 0.2$. What was the effect?

- (i) The graph stretched vertically by a factor of 5 because the y -coordinates were multiplied by 5.
- (ii) The graph stretched horizontally by a factor of 5 because the x -coordinates were multiplied by 5.
- (iii) The graph compressed vertically by a factor of 5 because the y -coordinates were divided by 5.
- (iv) The graph compressed horizontally by a factor of 5 because the x -coordinates were divided by 5.

(C) Experiment with this graph by setting c and d to fixed values of your choice. Now change a and b . Does changing a and b have any effect on the coordinates of the vertex. If so, which coordinates are affected?

- (D) Increase the value of c by 3. What is the effect on the graph?
- (i) The graph shifted left 3 units.
 - (ii) The graph shifted right 3 units.
 - (iii) The graph shifted up 3 units.
 - (iv) The graph shifted down 3 units.
- (E) Decrease the value of d by 2. What is the effect on the graph?
- (i) The graph shifted left 2 units.
 - (ii) The graph shifted right 2 units.
 - (iii) The graph shifted up 2 units.
 - (iv) The graph shifted down 2 units.
- (F) Look at any graph of this function when $a = 3$ and then when $a = -3$ (everything else should be the same) What is the difference between the graphs? Why did this occur?
- (G) Look at any graph of this function when $b = 0.8$ and then when $b = -0.8$ (everything else should be the same). Is there any difference between the graphs? Give an explanation of the occurrence you notice between these two graphs.
- (H) What happens when $a = 0$ or $b = 0$ (or both)? Give algebraic explanations as to explain the occurrences under these circumstances.

2. Choose the tab $y = |x|$

(A) What are the only changes that can be done to this parent function to make the graph be something that is not V-shaped?

(B) Which of the values (a, b, c, d) , when changed cause the coordinates of the corner point to change? Explain which of the coordinates of the corner point change provided any change occurs.

(C) Which of the values (a, b, c, d) , has no effect on the coordinates of corner point? Give an explanation as to why this occurs. [This may take several observations and calculations on your part.]

(D) Explain how the graph transformation rules are confirmed by the occurrences seen in (B) and (C).