

# PRECALCULUS REVIEW

## PART 2: FUNCTIONS, POLYNOMIALS, RATIONALS, EXPONENTIALS, AND LOGARITHMS (OH MY).

In this review packet, problems marked with a \* indicate that a calculator may need to be used.

### (A) FUNCTIONS

For each of the following functions do the following.

- State the domain and range of the function.
- Sketch a graph of the function.
- Find the inverse of the function.

[Hint: To find the range of some of the functions, it may be easier if you find the inverse first.]

1.  $f(x) = 4x - 3$

2.  $f(x) = -\frac{3}{2}(x+4)$

3.  $f(x) = \frac{4-5x}{6}$

4.  $f(x) = \frac{2}{3}(x-1)^2 + 4$

5.  $f(x) = 2x^2 - 1$

6.  $f(x) = \frac{5-x^2}{4}$

7.  $f(x) = 5 - \frac{x^2}{4}$

8.  $f(x) = (x+3)(x-5)$

9.  $f(x) = \sqrt{x-5}$

10.  $f(x) = \sqrt{4x+7}$

11.  $f(x) = \sqrt{x^2+4}$

12.  $f(x) = 6 + \sqrt{x}$

13.  $f(x) = 4 - \sqrt{x+1}$

14.  $f(x) = 4 - \sqrt[3]{x+1}$

15.  $f(x) = 8^{2x+1}$

\*16.  $f(x) = e^{3x-5}$

\*17.  $f(x) = e^{3x} - 5$

18.  $f(x) = 4^{x+1} + 7$

19.  $f(x) = \log(x+4)$

20.  $f(x) = \log_7(2x) - 1$

\*21.  $f(x) = \ln(x-3)^2$

22.  $f(x) = \log_5(x-7)$

**(B) FUNCTION OPERATIONS AND NOTATION**

Given the following functions...

$$f(x) = 3x + 2 \quad h(x) = \log(x + 2)$$

$$g(x) = \sqrt{x - 9} \quad j(x) = x^2 + 5$$

Find the following. [Note: Some of the answers will be numeric and others will be expressions. Make sure that you know how to tell which type of solution is to be obtained.]

23.  $f(-4)$

24.  $g(t)$

25.  $(h + j)(8)$

26.  $(f \cdot g)(13)$

27.  $j(2a)$

28.  $2j(a)$

29.  $f(x + h)$

30.  $-4g(25)$

31.  $\left(\frac{j}{f}\right)(x)$

32.  $(f \circ g)(9)$

33.  $(j \circ h)(98)$

34.  $(h \circ j)(x)$

35.  $(f \circ h \circ g)(73)$

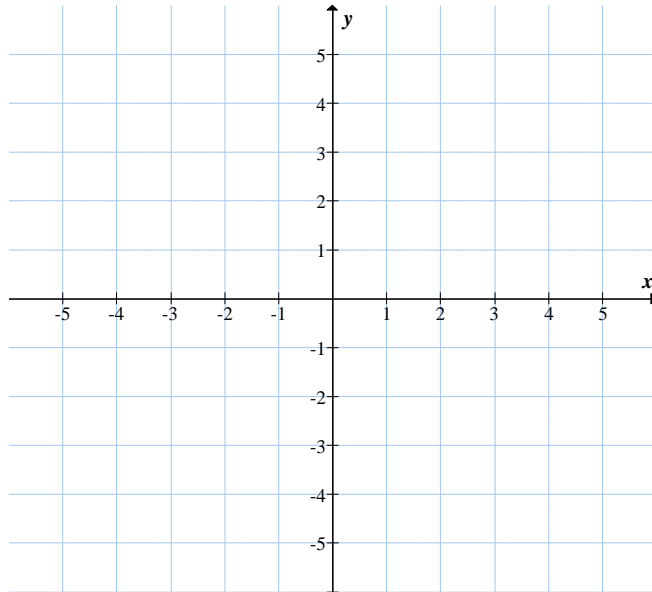
36.  $(j \circ g)(x)$

### (C) GRAPH TRANSFORMATIONS

37. Given  $f(x) = |x|$

(a) Fill in the table and graph the function in the spaces provided below.

$x$	$y = f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



(b) Now use graph transformation techniques to sketch the graphs of the following.

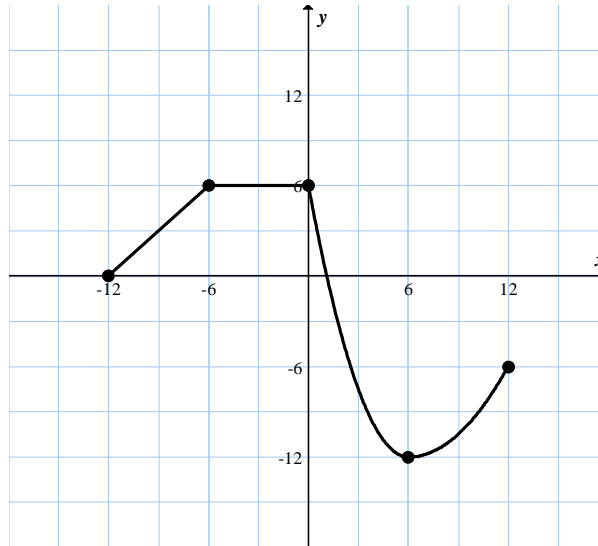
(i)  $g(x) = |x + 2|$

(ii)  $h(x) = |x| + 2$

(iii)  $j(x) = -3|x - 1|$

(iv)  $k(x) = \left|\frac{1}{2}x + 3\right|$

Given the graph below of the function  $y = g(x)$  below...



Use graph transformation techniques to graph the following functions. Make sure that you label your graph to indicate the scaling you used for your graph.

38.  $y = g(2x)$

39.  $y = -\frac{1}{3}g(x)$

40.  $y = g(-3x+4)$

41.  $y = 2g(x+3)$

42.  $y = 2g(x)+3$

43.  $y = \frac{2}{3}g(2x-1)$

44.  $y = \frac{1}{2}g(3x)-5$

45.  $y = -2g(2x+2)+2$

### (D) THE DIFFERENCE QUOTIENT

Compute and simplify the difference quotient for the following functions.

Note: The difference quotient of  $f(x) = \frac{f(x+h) - f(x)}{h}$

46.  $f(x) = 2x + 3$

47.  $f(x) = -4x$

48.  $f(x) = x^2$

49.  $f(x) = 3x^2 - 4x + 1$

50.  $f(x) = x^3 + 1$

51.  $f(x) = \frac{4}{x}$

52.  $f(x) = \sqrt{x}$

53.  $f(x) = \sqrt{3x - 2}$

### (E) PIECEWISE FUNCTIONS

Sketch the graphs of the following piecewise functions.

54.  $f(x) = \begin{cases} (x+2)^2 & \text{if } x \leq -2 \\ 4 - |x| & \text{if } -2 < x \leq 6 \end{cases}$

55.  $g(x) = \begin{cases} \frac{1}{2}x - 4 & \text{if } -4 \leq x \leq 2 \\ \sqrt{x+2} & \text{if } x \geq 2 \end{cases}$

56.  $h(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ |x| & \text{if } -1 \leq x \leq 2 \\ x^2 - 2 & \text{if } x > 2 \end{cases}$

57.  $k(x) = \begin{cases} \frac{2}{x} & \text{if } x \leq -1 \\ |x - 2| & \text{if } -1 < x < 5 \\ x^2 + 3x - 6 & \text{if } x \geq 5 \end{cases}$

**(F) POLYNOMIAL, RATIONAL, & RADICAL EQUATIONS**

**Find all REAL solutions for the following equations (if possible).**

**58.**  $2x^3 + 6x^2 - 5x = 15$

**59.**  $x^6 - 4x^5 - 5x^4 - 9x^2 + 36x + 50 = 5$

**60.**  $x^4 - 13x^2 + 40 = 4$

**61.**  $3x^6 + 21x^3 - 24 = 0$

**62.**  $\frac{x+3}{x-1} = 4$

**63.**  $\frac{x^2-4}{x+2} = 0$

**64.**  $\frac{x^2+2}{x^2+1} = 1$

**65.**  $\frac{x^2+6x-16}{x-2} = 0$

**66.**  $\frac{x^2+6x-16}{x-2} = 6$

**67.**  $|5x-4| = 8$

**68.**  $\sqrt{3x-2} = 7$

**69.**  $\sqrt{4x^2-10x+5} = x-3$

**70.**  $\sqrt{3x-11} = x-5$

**71.**  $\sqrt[3]{x^2-1} = 2$

**(G) RATIONAL FUNCTIONS**

**Graph the following rational functions. Be sure to indicate the equations of any vertical asymptotes, horizontal asymptotes, and oblique asymptotes depending on which you have. Also, where necessary, indicate the coordinates of any holes.**

**72.**  $f(x) = \frac{x+2}{x-3}$

**73.**  $f(x) = \frac{3x+1}{x+5}$

**74.**  $f(x) = \frac{2}{2x-5}$

**75.**  $f(x) = \frac{x^2-4}{x-2}$

**76.**  $f(x) = \frac{x-2}{x^2-4}$

**77.**  $f(x) = \frac{x^2+7x+10}{x^2-25}$

**78.**  $f(x) = \frac{x^2-x-6}{x-2}$

**79.**  $f(x) = \frac{x^2+2x}{x^2-4x-5}$

**(H) EXPONENTIAL & LOGARITHMIC EQUATIONS**

**Solve the following equations.**

**80.**  $3^{2x+1} = 27$

**81.**  $8^x = 4^{x^2-3}$

**\*82.**  $7^{2x-1} = 20$

**\*83.**  $e^{4x+5} = 8$

**\*84.**  $2^{3x+1} - 6 = 12$

**85.**  $6^{4x+3} + 8 = 2$

**\*86.**  $\ln(x+3) = 4$

**87.**  $\log_3(2x+1) = 2$

**88.**  $\ln(x) + \ln(2x-1) = \ln(15)$

**\*89.**  $\log_2(x^2 + 3x) - \log_2(x) = 11$

**90.**  $\ln(x+1)^2 = 0$

**91.**  $\log(2x+3)^3 = 9$

**Answer the following.**

**\*92.** The half-life of polonium ( $^{210}\text{Po}$ ) is 140 days. You are starting with 10 milligrams of polonium.

(a) If you start with 10 milligrams of polonium, how much will be left at the end of a year?

(b) How long will it take for 8 grams of the polonium to decay?

**\*93.** An insect colony grows exponentially from 200 to 2000 in 3 months. How long will it take for the insect population to reach 50,000?

**\*94.** Hydrogen-3 decays at a rate of 6% per year. Find its half-life.

**\*95.** The half-life of radium-88 is 1590 years.

(a) How long will it take for 10 grams to decay to 1 gram?

(b) If you started with a sample of radium-88, what percentage of the original amount would be left after 30 years?