PORTFOLIO PROJECTS
PRECALCULUS 12

RATIONALE:

As stated in the course description, the portfolio projects on the following pages are an opportunity for "students to demonstrate their ability to use a good problem-solving process." The purpose of these problems is to allow students to apply the knowledge they have obtained in this course to situations not necessarily found in the textbook. The reasons for doing so depend on the student's course level.

Weighted Courses: The problems for students in weighted courses are designed to further challenge them. It gives them the opportunity to develop the skills of "thinking outside the box," a skill that is needed as they progress towards Calculus in their senior year.

Non-weighted Courses: The problems for the students in the non-weighted courses are designed to allow students to review concepts from previous math courses to see how they can be applied to material they are currently learning and vice versa. These problems allow students to demonstrate their knowledge, even if they are students who have difficulty on traditional examinations.

In either case, the students will be judged on the work that they show as that is one way they will <u>communicate</u> the level of knowledge they have.

GENERAL DIRECTIVES:

1. This is meant to be an independent assignment. Although student collaboration is not only allowed, but encouraged, every student is responsible for obtaining and understanding the solutions to these problems. In order to determine this, there may be instances when the students will be questioned to see if they understand their solution.

2. These problems are meant for students to research and explore mathematics outside of outside of situations highlighted in their textbook. The students will have the following resources to aide them in solving the enclosed problems:

   (a) My classroom has various math textbooks.
   (b) Occasionally, class time will be devoted to checking the student's progress and addressing difficulties. (Note: the amount of class time for this will be limited.)
   (c) Since many of the problems come from mathematical organization websites, students may be able to find similar problems and explanations on the internet.
   (d) Many of the projects are investigations in which students will utilize data that they gather.
   (e) I am often available for tutoring outside of the classroom.

3. **WORK MUST BE SHOWN ON ALL PROBLEMS. CORRECT BUT UNSUPPORTED SOLUTIONS WILL RECEIVE LITTLE OR NO CREDIT.**

<u>REMINDER</u>: THIS IS PART OF THE PORTFOLIO, WHICH IS A YEAR-LONG PROJECT. THEREFORE, STUDENTS SHOULD BE WORKING ON THESE PROBLEMS THROUGHOUT THE ENTIRE YEAR.

I WILL ALSO BE GIVING DIRECTIVES AS TO WHEN CERTAIN PROJECTS SHOULD BE STARTED AND FINISHED AS WE PROGRESS THROUGH THE MATERIAL NECESSARY TO COMPLETE THESE PROBLEMS.

INTRODUCTION TO PARAMETRIC EQUATIONS

PART 1: BASICS

Up to this point, we have been graphing equations in function notation $y = f(x)$. This format indicates that *y* is a function of *x* and that *x* is the independent variable while *y* is the dependent variable (the value of *y* depends on the value of *x*).

There are circumstances however, when the values of *x* and *y* are both dependent upon a third variable. Such a situation makes use of **parametric equations**.

PARAMETRIC FORMAT

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

where *x* and *y* are both functions of the variable *t* which is called the **parameter**.

Now let's work through an example of how to produce a graph from parametric equations.

EXAMPLE: Draw the graph of the parametric equation given below.

$$\begin{cases} x = t^2 + 3 \\ y = \frac{1}{2}t - 4 \end{cases} \quad -3 \le t \le 3$$

First, construct a table with the variables *t*, *x*, and *y*. The values of *t* are chosen at random since it is the independent variable. In this case, we will choose values between −3 and 3 as indicated above. Such a table has been started on the following page.

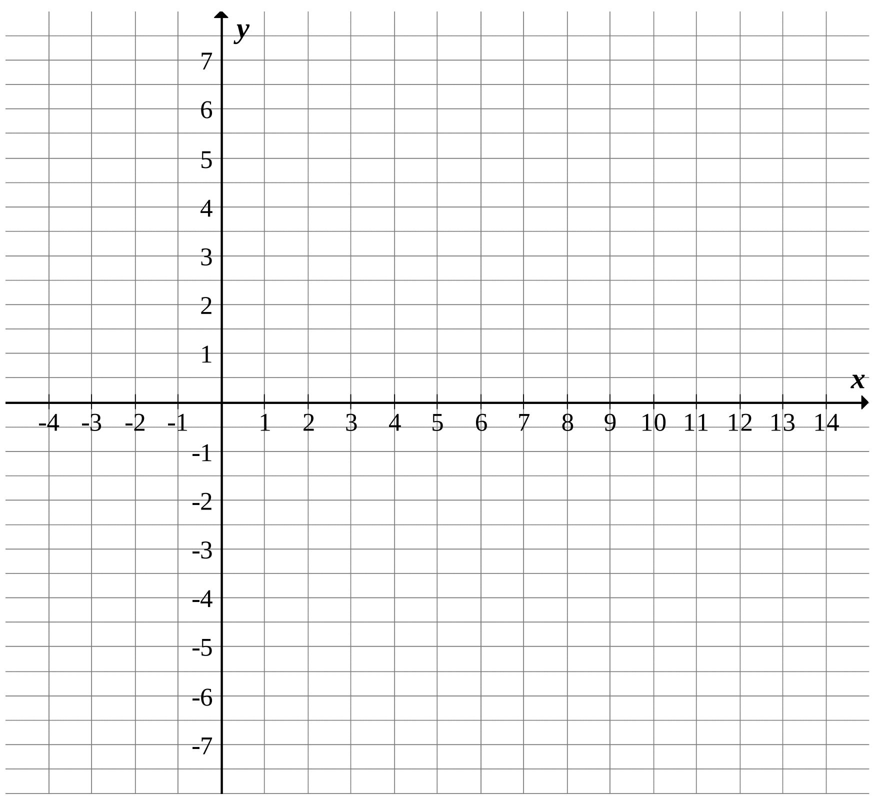
t	*x*	*y*
–3	12	–5.5
–2		
–1		
0		
1		
2		
3		

Now that we have a table we can draw the graph. There are two things that you need to keep in mind as you do so.

(1) Graphs are only of the *x*- and *y*-coordinates only. The values of *t* do not appear on such a graph.

(2) Parametric graphs are typically drawn as **finite** graphs. That is they are defined for, and as a result begin and end, at specific values of *t*.

Draw the graph below.



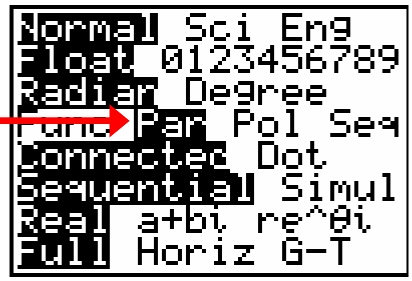
PART 2: USING THE TI-83 PLUS CALCULATOR

Now let's see if the graph that you drew in Part 1 is correct. The TI-83 Plus calculator is capable of graphing parametric equations as well. The procedure shown on the following pages will show you how.

First, put your calculator in parametric mode.

- Press [MODE] and you will see the window below appear.

This indicates that the calculator is in Parametric mode.

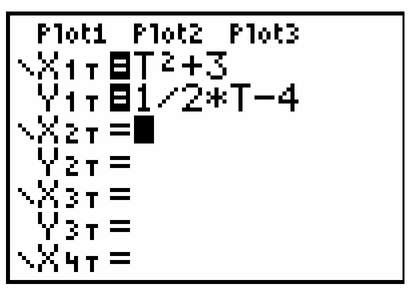


- Choose **Par** by moving the cursor over it and pressing [ENTER].

Now we will enter the parametric equations from the example in Part 1.

- Press [Y=]
- Enter the *x* equation next to "\X_{1T} = "
- Enter the *y* equation next to "Y_{1T} = "
- Your calculator window should now look like the picture below.

Note: Pressing [X,T,Θ,*n*] is the easiest way of entering "T" in parametric mode.



Before graphing the function, the window needs adjusting.

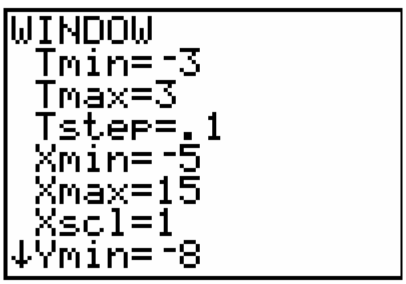
- Press [WINDOW]

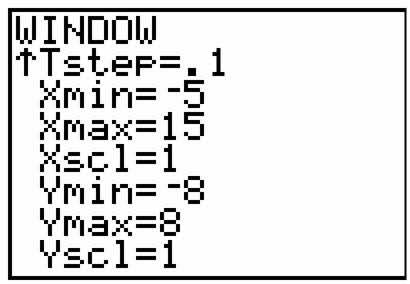
- The window is pretty much the same as the one that you have seen when graphing functions on the calculator except there are 3 new things.

T_{min} = the minimum value for T.
T_{max} = the maximum value for T.

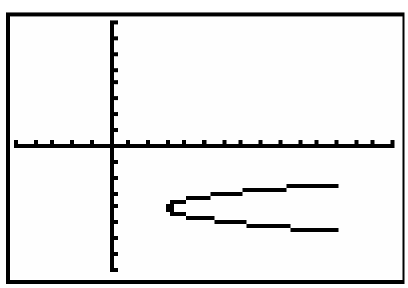
T_{step} : The calculator graphs functions like we have done in using the table. It starts with T_{min}, calculates the values for X and Y, plots the point, and then moves on to another value for T. T_{step} tells the calculator how much to move when going to the next value of T to repeat the process. The smaller the value for T_{step}, the more accurate the graph, however, the slower the calculator draws the graph. Generally, it is best to keep this value at about 0.1.

- Set up the window as shown in the pictures below. Since $-3 \le t \le 3$, $T_{min} = -3$ and $T_{max} = 3$.





- Now press [GRAPH] and the graph should look like the one below.



Does the graph you drew in Part 1 look like this graph?

Now that you have a graph you can also check the calculations using the calculator.

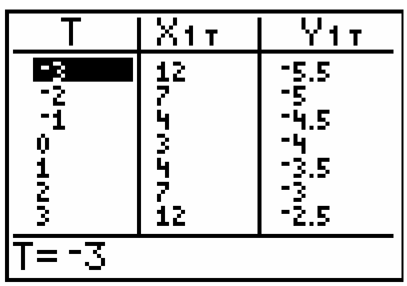
- Press [2nd] then [WINDOW] to access the TABLE SETUP window shown below.

TblStart = The first value of the independent variable (in this case, *t*) you want to see on the table.

△Tbl = The interval between the *t*-values in the table.

- Make sure that your settings for the TABLE SETUP window are the same as those shown in the figure above.

- Now press [2nd] then [GRAPH] to pull up the table.



Does this table match the one you made in part 1?

Now it's time to practice some of these skills.

EXERCISES

Graph the following parametric equations for the given intervals of *t* on a separate piece of graph paper. Check your answers using your TI-83 Plus.

1. $\begin{cases} x = 2t \\ y = 3t \end{cases} \quad -6 \le t \le 6$

2. $\begin{cases} x = 5 \\ y = 2t \end{cases} \quad -5 \le t \le 5$

3. $\begin{cases} x = t \\ y = t^2 + 4t + 5 \end{cases} \quad -6 \le t \le 6$

4. $\begin{cases} x = \left|\frac{1}{2}t\right| \\ y = t \end{cases} \quad -10 \le t \le 10$

5. $\begin{cases} x = 4t^2 - 16 \\ y = t^2 - 9 \end{cases} \quad -3 \le t \le 3$

6. $\begin{cases} x = 3t^2 + 9 \\ y = 2t^2 + 6 \end{cases} \quad -2 \le t \le 2$

7. $\begin{cases} x = \sqrt{25 - t^2} \\ y = \sqrt{16 - t^2} \end{cases} \quad -4 \le t \le 4$

8. $\begin{cases} x = \sqrt{9 - t^2} \\ y = t^2 - 4 \end{cases} \quad -3 \le t \le 3$

9. $\begin{cases} x = 2 + t - 2t^2 - t^3 \\ y = 2 - t - 2t^2 + t^3 \end{cases} \quad -1 \le t \le 1$

10. $\begin{cases} x = (t + 2)(t - 1)(t - 3) \\ y = t \end{cases} \quad -3 \le t \le 4$

<u>PART 3</u>: WRITING PARAMETRIC FUNCTIONS IN FUNCTION FORMAT

Due to the fact that it is often the case that we want to find information about particular points along the graph produced. We usually use the calculator to do this (although we will eventually learn to do it ourselves), but the calculator cannot find minima, maxima, zeros or anything like those in parametric mode.

Therefore, it is important to know how to take a parametric function (a parametric equation that produces a graph that passes the Vertical Line Test) and convert it into function notation. This is done by eliminating the variable *t* through a number of substitutions. That is why this procedure is called **eliminating the parameter**.

<u>EXAMPLE</u>: Convert the following to function format.

(A) $\begin{cases} x = 2t + 4 \\ y = 6t - 3 \end{cases}$ **(B)** $\begin{cases} x = t - 2 \\ y = t^2 + 3t - 6 \end{cases}$

<u>SOLUTIONS</u>: Note the procedure used.

(A) <u>Step 1</u>: Solve the *x*-equation for *t*.

$$x = 2t + 4$$
$$2t = x - 4$$
$$t = \frac{x-4}{2} = \frac{1}{2}x - 2$$

<u>Step 2</u>: Substitute for *t* in the *y*-equation. $y = 6\left(\frac{1}{2}x - 2\right) - 3$

Technically, this is a suitable answer. However, it is possible to simplify the answer a bit.

<u>Step 3</u>: Simplify.

$$y = 3x - 12 - 3$$
$$y = 3x - 15$$

Now let's solve part (B).

(B) Step 1: Solve the *x*-equation for *t*.

$$x = t - 2$$
$$t = x + 2$$

Step 2: Substitute for *t* in the *y*-equation. $y = (x+2)^2 + 3(x+2) - 6$

Again, this is a suitable answer. However, it is possible to simplify the answer a bit. Some of this may not make sense if you do not remember how to FOIL or know what that is (depending on your Algebra 1 experience). So, if you give an answer you can stop at the answer in step 2.

Step 3: Simplify.

$$y = x^2 + 4x + 4 + 3x + 6 - 6$$
$$y = x^2 + 7x + 4$$

EXERCISES

Convert the following to function format.

1. $\begin{cases} x = 2t \\ y = t - 1 \end{cases}$

2. $\begin{cases} x = t + 3 \\ y = 3t \end{cases}$

3. $\begin{cases} x = t \\ y = 3 - 2t \end{cases}$

4. $\begin{cases} x = 4t + 1 \\ y = t + 5 \end{cases}$

5. $\begin{cases} x = t + 5 \\ y = 4t + 1 \end{cases}$

6. $\begin{cases} x = \frac{1}{3}t \\ y = t^2 - 1 \end{cases}$

7. $\begin{cases} x = 0.1t \\ y = t^2 \end{cases}$

8. $\begin{cases} x = 4 - t \\ y = 2t^2 - 4t + 9 \end{cases}$

PROJECTILE MOTION

PART 1: RESEARCH

READING YOUR TEXTBOOK:

Read pages 759-763 of your textbook (starting with Example 5) and complete the following statements.

Given a projectile is fired from a height of k above the ground [that is position $(0, k)$]

(1) The y-axis represents the __________________ of the object at time t and the x-axis represents the ______________________________ of the object. All distances in this section are in feet.

(2) The horizontal position of the object at time t (in seconds) is given by:

$$x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

The vertical position of the object at time t (in seconds) is given by:

$$y = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

v represents ____________________________

θ represents _______________________________

(3) Since the position of the object is given by separate equations for x and y, we call the format of this function to be _________________________________.

GOING BEYOND THE BOOK:

There is a problem using PARAMETRIC MODE on your calculator. You can use it to obtain the graph of the path described by the two functions BUT, most problems ask you to find where the object hits the ground (or reaches some other height) or to find the maximum height of the object. Your calculator has features to find such things, unfortunately, it cannot be done in PARAMETRIC MODE. It can, however, find maximums and roots (that is when the height is 0 and therefore crosses the x-axis) in FUNCTION MODE.

Therefore, we are going to want to use a formula of a single function (in terms of x and y only) instead of a set of parametric functions. This can be done by following the following steps.

(1) Take the equation for x you wrote in step 2 and solve it for t. Write that equation below.

(2) Now using the equation you found in step 4, substitute for t in the equation for y you found in step 2. Simplify the equation, but make sure y is left alone on the left. Write that equation below:

$$y = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

[Remember: v, θ, and k are constants.]

Now we have the equation of a function that would produce the same graph the two parametric functions would.

<u>PART 2</u>: EXAMPLE & TUTORIAL

The following instructions have been written for use on the TI-83 Plus graphing calculator (a TI-84 should use the exact same steps).

<u>The Problem</u>: A ball is thrown from a height of 5 feet above the ground with an initial velocity of 60 feet/second at an angle of 50° with the horizontal.

(A) Graph the ball's path.

(B) What is the maximum height of the ball?

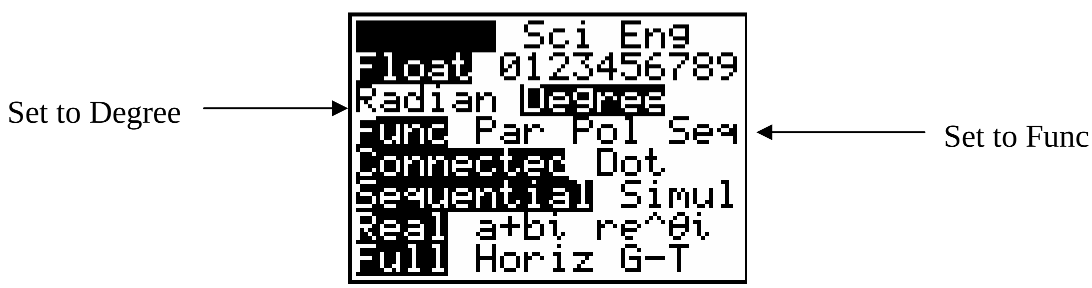
(C) When and where does the ball hit the ground?

<u>Solution</u>:

(A) GRAPHING THE FUNCTION

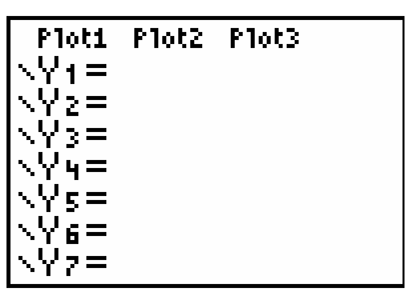
☆ First we make sure that the calculator is set to the correct modes. We are going to use PARAMETRIC functions and the angles are given in DEGREES we want those modes set on the calculator.

Press [MODE] to access the MODE menu (shown below)



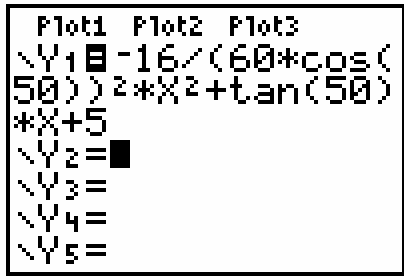
☆ Next we will enter the equations according to the formulas given in the notes.

Press [Y=] and you will see the screen below on your calculator.

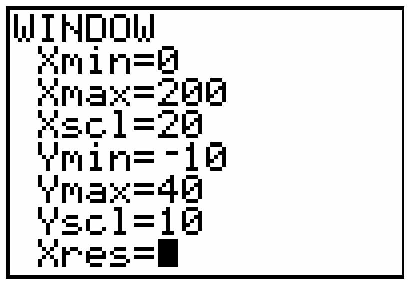


☆ From our results of Part 1 we will enter,

“\Y1 = -16/(60*cos(50))^2 * X^2 + tan(50) * X + 5”

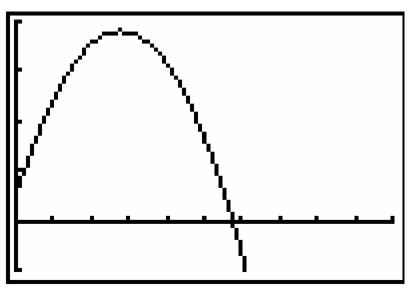


☆ Before graphing the function, we should adjust the graphing window. Press [WINDOW] and set the window settings as shown below.



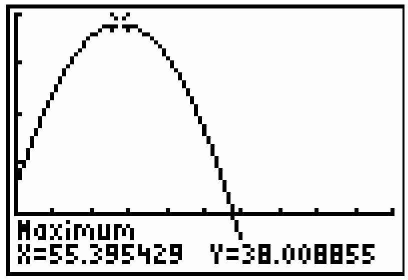
NOTE: These setting will not always be the ones to use. YOU will have to adjust them according to the situation you are trying to model.

☆ Now press [GRAPH] to graph the function. The function appears below. (You always want to have a picture where the top of the path can be seen and where the graph crosses the *x*-axis.)



(B) FINDING THE MAXIMUM HEIGHT

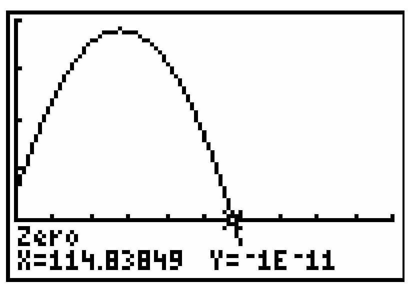
By this time, you have already learned how to find the maximum, minimum, and roots of a function using your calculator. As seen in the figure below, the function does reach a maximum height. Find the coordinates of the maximum. You should get something like the screen below if you did it correctly.



Since the y-value represents the height of the object, the maximum height is about **38 ft**.

(C) WHEN AND WHERE THE BALL HITS THE GROUND

The ball hits the ground when the height = 0. Therefore, we would like to find the root of the function. Again, you we have already learned how to find roots using the calculator. If you do that properly, you should get a screen that looks like the one below.



Note the "1E-11" for the *y*-value when we expected 0. This sometimes happens when using this feature depending on various settings. When you see a number that is so small that it has to be written in scientific notation, it can be safely assumed to be 0.

Anyway, we can see that the value of *x* is 114.8384…, which means that the distance downfield the ball lands is about **115 feet**. However, we still need to find the "when" which means that we have to find *t*.

Remember from #4 of part 1, $t = \dfrac{x}{v\cos\theta}$, so to find the time we compute the following.

$$t = \frac{114.8384}{60\cos(50)} \approx 2.98 \text{ seconds}$$

So the ball lands about **115 feet downfield about 3 seconds later**.

EXERCISES:

- ✯ Complete #37-39 & 41 on page 764
- ✯ Complete the exercise below.

A football is kicked from the ground at an initial velocity of 75 feet/second.

(A) Use techniques from this investigation to complete the following table.

ANGLE OF ELEVATION, θ (degrees)	MAXIMUM HEIGHT OF FOOTBALL (ft)	MAXIMUM DISTANCE FOOTBALL TRAVELS (ft)
10°		
20°		
30°		
40°		
50°		
60°		
70°		
80°		
90°		

(B) Relating the angle of elevation to the maximum heights for the object, at what angle should the football be kicked?

(C) Relating the angle of elevation to the maximum distances for the object, at what angle should the football be kicked? [To answer this question, you may want to run a regression to aide you in doing so.]

HOW HIGH IS ST. JOE'S HIGH?

MATERIALS NEEDED:

Protractor Elevation Finder
(will have to be made from a protractor, string & weight)

2 Tape measures

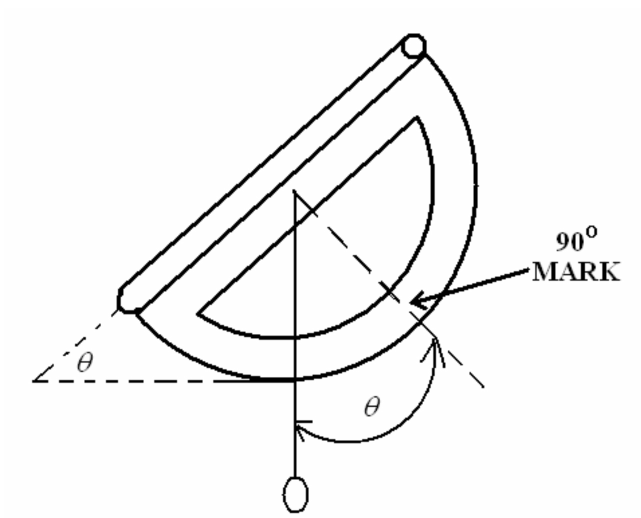
STUDENTS SHOULD WORK IN TEAMS OF 4 TO COMPLETE THIS PROJECT

PART 1: FIND THE HEIGHT OF THE FLAGPOLE IN THE FRONT LAWN

1. You will stand at a given distance from the flagpole as outlined in the table

2. Standing at the given distance, one person, the surveyor, will look through the straw on the Protractor Elevation Finder (once you assemble one) and sight the top of the flagpole.

3. A second person will measure the angle of elevation, θ, given by the string on the Protractor Elevation Finder (see the figure below)

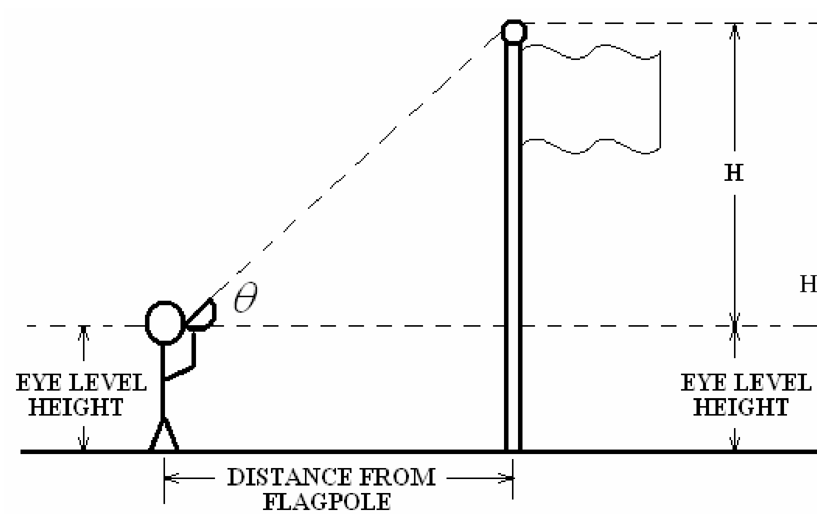


4. A third person will measure the height from the ground to the surveyor's eye level.

5. A fourth person will record the measurements in the first 3 columns in the table below. [You should use 3 different viewpoints for each given distance from the flagpole.]

DISTANCE FROM FLAGPOLE (ft)	ANGLE OF ELEVATION (degrees)	EYE LEVEL HEIGHT (ft)	HEIGHT OF FLAGPOLE (ft) [will have to be calculated]
20			
20			
20			
30			
30			
30			
50			
50			
50			

6. To fill in the fourth column, "HEIGHT OF FLAGPOLE," you need to compute the amount. Use the illustration and formulas below to compute the flagpole's height from each distance and viewpoint.



$$\tan\theta = \frac{H}{\text{DISTANCE FROM FLAGPOLE}}$$

$$H = \text{DISTANCE FROM FLAGPOLE} \cdot \tan\theta$$

HEIGHT OF FLAGPOLE = H + EYE LEVEL HEIGHT

The computed heights may differ slightly due to measurement errors, calculation and rounding errors, and other factors. Therefore, taking multiple measurements helps eliminate those errors.

7. Once you have the 9 heights calculated, average them together and this will be your estimate of the flagpole's height.

According to your readings and calculations, what is the height of the flagpole in the front lawn?

<u>PART 2</u>: FIND THE HEIGHT OF ST. JOSEPH HIGHSCHOOL

You are now going to use the same procedures like those used in measuring the flagpole to measure the height of St. Joseph High School. As you make your measurements you will only be measuring from street level (which means you will not include Patterson Hall) and from either the front lawn and/or the SJHS parking lot. This will eliminate some of the variables due to inclines of the streets and such.

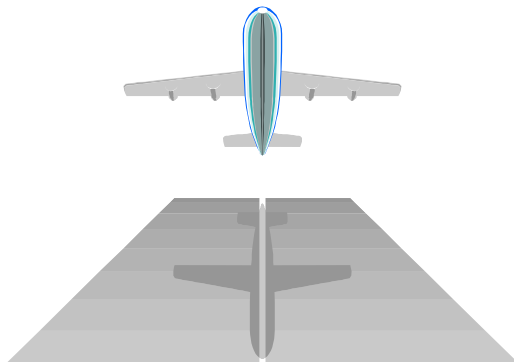
Complete the table below and estimate the height of SJHS. (Again, for each distance from the building you should have three different viewpoints.]

DISTANCE FROM SJHS (ft)	ANGLE OF ELEVATION (degrees)	EYE LEVEL HEIGHT (ft)	HEIGHT OF SJHS (ft) [will have to be calculated]
30			
30			
30			
60			
60			
60			
100			
100			
100			

THE PHRASE, "FINAL APPROACH," DOESN'T SOUND TOO PLEASANT.

When approaching an airport pilots learn that the best angle of decent for the final approach is 3 degrees. When the plane reaches the inner marker of a particular airport, which marks the beginning of the final approach zone, it is supposed to be at 500 ft altitude to begin the final approach.

(1) Based on the above information, how far must the final approach/landing zone be for the plane to safely land?



(2) If the airplane is at 1500ft when it reaches the outer marker which is 2 miles from the inner marker, what does the angle of decent need to be for the plane to reach the 500 ft altitude when it reaches the inner marker?

SKIING IN VERMONT

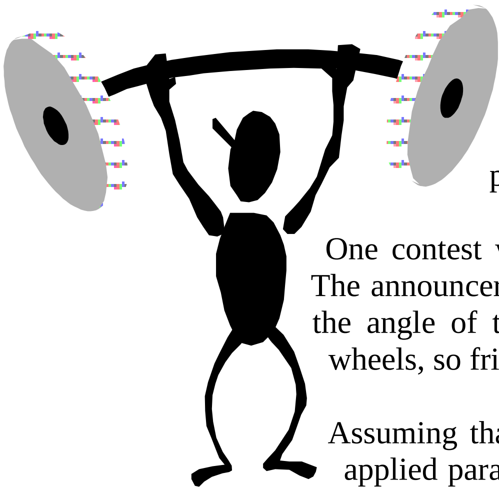
You are on a cross-country skiing trip in Vermont, you are being told the directions that you will be traveling on this outing by the ski instructor.

“Starting at the lodge, we will head 40 degrees north of east for 0.5 km,” says the instructor. He continues, “Then, we will go 50 degrees north of west for 1.2 km and then due west for 2 km. Last we will head…” Just then a strong wind picks up and the blowing in your ear keeps you from hearing what he is saying. The wind dies down for you to hear the instructor finish his statement, “…and then we will be back at the lodge.”

You figured that it was not necessary to worry about not hearing the instructor as you would be following him anyway. But as you and your group finish the third leg of the trip, a snow storm hits and you fall into a ditch. You take cover to find after the storm subsides that the rest of the group is gone and the snow covered their tracks. So, how far are you from the lodge and in what heading will you need to travel to get there?

PROJECTIONS

THE WORLD'S STRONGEST MAN



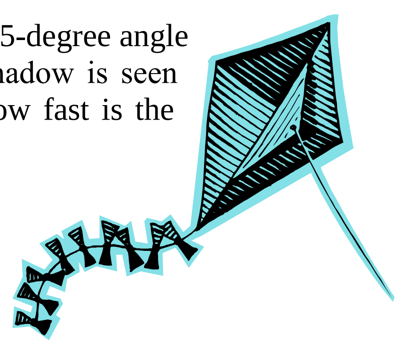
While watching TV a few weeks ago, I saw a competition for the world's strongest man. This competition features several events, like throwing barrels over walls, pulling heavy objects, and putting heavy stones on steps.

One contest was to push a piano up a ramp in the fastest time. The announcer said that the piano weighed about 318 pounds, and the angle of the incline was 20 degrees. The piano is also on wheels, so friction is negligible.

Assuming that the man will push the piano so the force will be applied parallel to the surface of the ramp, how much force will he need to apply to move the piano?

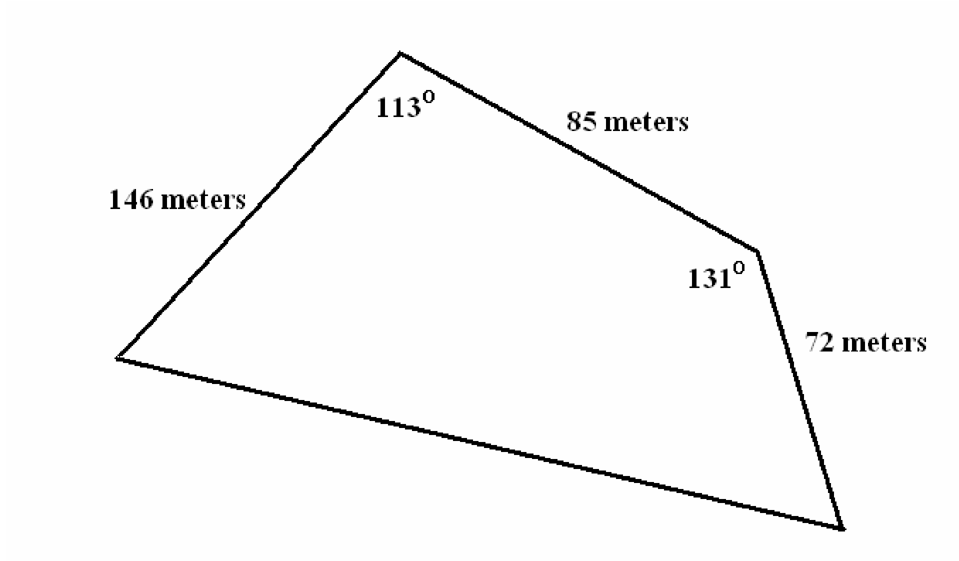
LET'S GO FLY A KITE

A kite is being flown over a hill. The kite moves at a 15-degree angle from horizontal at a speed of 20 mph. The kite's shadow is seen moving down a hill that has a 12-degree incline. How fast is the kite's shadow moving?



HOW LARGE IS THE YARD?

In order to apply for a building permit, your parents had to get their property surveyed. They live on a lot shaped as it is shown below (not drawn to scale).



The surveyors took measurements of three sides and two included angles. From this information they were able to determine the area of the yard.

The surveyors told your parents that the area was 2.5 acres. Your parents, on the other hand, did not agree. They believe that they have more land than that, so they ask you to use your mathematical skills to determine the area of their yard.

Did the surveyors calculate the area of the yard correctly? Explain your reasoning.

MORE TRIGONOMETRIC GRAPHS

Although these problems can be done with your calculator, it will be better for you to do them using **Graph 4.3** since you can print out your results and use that as your drawing. If you use your calculator you will have to draw the graphs by hand.

For #1-4, draw the graph of the function. Make sure all waves can be seen in their entirety and that you show <u>at least two full periods</u> of the graph. Determine what the period is of the function (consult Example 2 on p. 512 of your textbook for assistance in calculating the periods of such functions).

1. $f(t) = (5\sin(2t))(\cos(5t))$

2. $f(t) = 3\sin(2t) + 6\cos(3t)$

3. $f(t) = 6\cos(2t-1) - 4\sin(3t+2)$

4. $f(t) = 6\sin(4\pi t) - 4\cos(6\pi t)$

For #5-8, draw graph the following functions for the range of *t* given. Make sure that all waves can be seen clearly and in their entirety by adjusting the *y*-axis accordingly.

5. $f(t) = -\sin\left(\frac{2}{t}\right)$

$-\pi \le t \le \pi$

6. $f(t) = \frac{1}{t}\sin(t)$

$-4\pi \le t \le 4\pi$

7. $f(t) = \ln|\cos t|$

$-2\pi \le t \le 2\pi$

8. $f(t) = t\cos t$

$-8\pi \le t \le 8\pi$

EXPONENTIAL APPLICATIONS

It should be noted, using procedures that were demonstrated in the lab EXPONENTIAL FUNCTION INVESTIGATION will be extremely useful in helping you answer the two questions in this portfolio project.

You will need to provide all work; tables, formulas used, etc.; as part of your solution.

THESE ARE THE VOYAGES OF THE STARSHIP ENTERPRISE

The dilithium chamber of the Starship Enterprise produces radiation. In order for the crew to work around it, an electromagnetic damping field is used to keep the radiation from causing injury to the engineering crew. As long as the radiation level in the engineering section of the ship remains at 10 rad (radiation unit) or less it is safe, above that damage can occur depending on the amount of exposure. When it reaches 50 rad, it causes sickness upon short exposure. At 100 rad, the radiation can start causing permanent damage and at 1000 rad it is deadly.

The damping field on the ship breaks down. Using robots, the crew is able to fix the damping field as it reaches 1000 rad. They cannot enter engineering until the radiation reaches a safe level. The dilithium chamber generates radiation that increases the level by 10 rad of every minute. The damping field emits an electromagnetic blast every 5 minutes that reduces the radiation by 80%. Will this allow the radiation level to reach a safe level?

Would a setting on the damping field causing the electromagnetic blast to be emitted every 3 minutes be better? Explain your answer.

PAYING OFF YOUR CREDIT CARDS…PRICELESS

You currently owe $10,000 on your credit card. This credit cards has a percentage rate that is compounded monthly. The interest rate on your credit card is currently 23.5% A.P.R.

(A) The minimum payment for this company is 2.5% of the balance after the interest is applied or $20 whichever is higher.

(i) What is this month's minimum payment?

(ii) How long would it take you to pay off the credit card balance if each of your monthly payments was the value you calculated above?

(iii) How long would it take you to pay off your credit card is you made the minimum payment each month? [Remember, under these circumstances the minimum payment will change each month as the balance changes.]

(B) You decide to go into a credit counseling program. Which of the two listed below would be the better of the two? [Point to consider; what would make one of these programs better than the other?]

PROGRAM 1: Reduces your current balance by $2000. Interest is reduced to 20% A.P.R. You make $200 payments per month until the remaining debt is paid off.

PROGRAM 2: Cuts your interest rate in half. You make $150 payments per month until the debt is paid off.

YOUR FINAL PORTFOLIO PROJECT QUESTION

The length of a toothpick is 2.5 inches long. (Yeah, I actually measure one…I have an exciting life!) You have 12 of them. Now the question that I am asking will sound easy, but is far more complicated than it may first appear. How would you arrange these 12 toothpicks in order to enclose the greatest possible area? What is that greatest possible area?

Your submitted solution should do three things:

(1) Convince me that the solution you obtained is correct and that you know why it is correct. This can be done by stating mathematical facts and/or showing calculations

(2) Shows all calculations involved. Even though the calculations for the area of the final figure are necessary, there can and probably should be other calculations.

(3) Gives a scaled drawing of the final answer, including angle measurements. You could also give an actual toothpick creation of the figure if you desire (or if it is even practical).