LAB: TRIGONOMETRIC FUNCTIONS, ROTATIONS & RADIAN MEASUREMENT

In the previous lab, *INTRODUCTION TO RADIAN MEASUREMENT*, we explored what radian measurement was. We have also discussed in previous classroom work the trigonometric functions of angles (in degree measurement) and how they were defined. Now we will be combining these two ideas together to do the following:

1. Define the trigonometric functions for angles over 90°
2. Find the values of the trigonometric functions for specific angles in radian measurement and find patterns that exist in those values.

PART 1: DEFINITION OF TRIGONOMETRIC FUNCTIONS

(1) Open the webpage entitled *CIRCLES & RADIAN MEASUREMENT* that we used in the last lab investigation found at the address below.

http://atsorren.freewebpages.org/GENERAL/RADIANS/radian.html

Once you open the page, click on the box "Show Coordinates." Notice that the coordinates of the points A (on the red, unit circle), B (on the blue circle) and C (on the green circle) now appear. These points mark the radius of rotation of the terminal ray.

(2) Use the applet to choose 4 angles (one from each coordinate) to complete the information in the first 5 columns of Table 1 on the next page (some of the information is already filled in). As you fill in this table, be aware of the following.

$x =$ The x-coordinate of the endpoint on the radius of rotation

$y =$ The y-coordinate of the endpoint on the radius of rotation

$r =$ The measurement of the radius of rotation (this is ALWAYS positive)

TABLE 1

QUADRANT	ANGLE, θ [in degrees]	r	x	y	$\frac{x}{r}$	$\frac{y}{r}$	$\frac{x}{y}$	$\frac{r}{x}$	$\frac{r}{y}$	$\frac{y}{x}$
I		1								
		2								
		3								
II		1								
		2								
		3								
III		1								
		2								
		3								
IV		1								
		2								
		3								

(3) Use the information from the first five columns and your calculator to complete the information for Table 1.

(4) Aside from slight differences due to rounding done by the applet, are the values found in the last six columns the same for each angle?

(5) Use the same angles that you used in Table 1 and your calculator to complete Table 2 on the next page.

TABLE 2

ANGLE, θ [in degrees]	$\sin\theta$	$\cos\theta$	$\tan\theta$

(6) Based on your observations and comparing values you obtained in Tables 1 and 2, write the formulas for the following trigonometric functions in terms of *x*, *y*, and/or *r*.

We only need to know the formulas for these trigonometric functions since the other three are just the reciprocals of these three.

$$\sin\theta =$$

$$\cos\theta =$$

$$\tan\theta =$$

(7) Explain, in your own words, why the unit circle is what is usually used to define these trigonometric functions, particularly sine and cosine.

(8) When working with the unit circle, the *x*-coordinate of the endpoint of the radius of rotation is equal to which trigonometric function?

(9) When working with the unit circle, the *y*-coordinate of the endpoint of the radius of rotation is equal to which trigonometric function?

(10) Complete the statement below:

Given a point P where the terminal ray of angle θ intersects the unit circle, the coordinates of P are given by:

P = (_____θ, _____θ)

(11) Use the information you gathered in this lab thus far to prove the trigonometric identity below:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

[This should be shown using algebraic equations and steps.]

PART 2: PATTERNS AND RADIAN MEASUREMENT

(1) In class notes, we discussed reference angles and how they are calculated. Complete column 2 of Table 3 on page 9 based on what you have already learned about reference angles.

(2) In the lab *INTRODUCTION TO RADIAN MEASUREMENT*, as well as in class, we determined the radian measurements of some particular angles. Complete column 3 of Table 3 on page 9 based on what you have already learned about radian measurement.

(3) Based on your observations, complete the statements below.

(i) *Angles written in radian measurement with a reference angle of* ***0°****, when written in terms of* π *and in lowest terms have a denominator equal to* _____.

[This is the same as not having a denominator at all.]

(ii) *Angles written in radian measurement with a reference angle of* ***30°****, when written in terms of* π *and in lowest terms have a denominator equal to* _____.

(iii) *Angles written in radian measurement with a reference angle of* ***45°****, when written in terms of* π *and in lowest terms have a denominator equal to* _____.

(iv) *Angles written in radian measurement with a reference angle of* ***60°****, when written in terms of* π *and in lowest terms have a denominator equal to* _____.

(v) *Angles written in radian measurement with a reference angle of* ***90°****, when written in terms of* π *and in lowest terms have a denominator equal to* _____.

(4) Use the applet on the webpage and the unit circle to complete columns 4 & 5 of Table 3 on page 9. You are to use the **EXACT** values by converting the decimals values you obtain from the applet into the values indicated in the figure below. [Note: Not all of the results will be those seen in this figure.]

$$\sqrt{2} \approx 1.41421 \qquad \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.70711$$

$$\sqrt{3} \approx 1.73205 \qquad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.57735 \qquad \frac{\sqrt{3}}{2} \approx 0.86603 \qquad \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \approx 1.15470$$

(5) Use your results from Part 1 (#11 to be exact) or your calculator to complete column 6. Again use **EXACT** values.

(6) Based on all of your observations up to this point, complete the statements below with either POSITIVE or NEGATIVE.

(i) If θ is an angle with the terminal ray in Quadrant I, then:

$\sin\theta$ is _____ $\cos\theta$ is _____

$\tan\theta$ is _____

(ii) If θ is an angle with the terminal ray in Quadrant II, then:

$\sin\theta$ is _____ $\cos\theta$ is _____

$\tan\theta$ is _____

(iii) If θ is an angle with the terminal ray in Quadrant III, then:

$\sin\theta$ is _____ $\cos\theta$ is _____

$\tan\theta$ is _____

(iv) If θ is an angle with the terminal ray in Quadrant IV, then:

$\sin\theta$ is _____ $\cos\theta$ is _____

$\tan\theta$ is _____

(7) Provide a detailed explanation (that means formulas should be used in your explanation) to describe why the observations you made in #6 occur.

(8) Looking at only the angles with a reference angle of 30°, what similarities do you notice about the values of sine, cosine, and tangent of these angles? What are the differences?

(9) Do the observations you noticed in #8 occur for the angles in Table 3 with reference angles other than 30°?

(10) Now complete the last 3 columns of Table 3. Since these angles are simply the reciprocals of the first 3 trigonometric functions, this should be quite easy at this point. [Please note, taking a reciprocal never, ever causes a sign change in a value.]

(11) Notice that on Table 3, there was a part that was shaded. It is my contention that in order to know all of the information on Table 3, all you would have to do is memorize the shaded portion of the table. In your own words, explain how the findings of this lab investigation justify this statement.

YOU ARE NOW REQUIRED TO KNOW ALL OF THE INFORMATION ON TABLE 3

TABLE 3

ANGLE, θ [in degrees]	REFERENCE ANGLE, α [in degrees]	ANGLE, θ [in radians]	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°								
30°								
45°								
60°								
90°								
120°								
135°								
150°								
180°								
210°								
225°								
240°								
270°								
300°								
315°								
330°								
360°								