LAB: INTRODUCTION TO TRIGONOMETRY

Materials needed:

- Scientific Calculator
- Computer with internet access
- RIGHT TRIANGLE web page (which can be accessed at the link below)

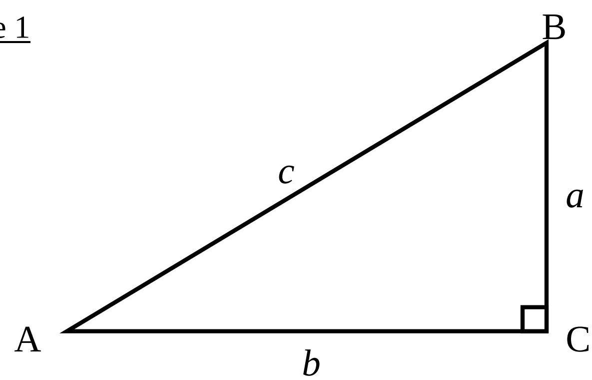
http://atsorren.freewebpages.org/GENERAL/RTTRIANGLE/right-triangle.html

PART 1: LAB PREPARATION

BASIC TERMINOLOGY

Given the right triangle as marked below:

Figure 1



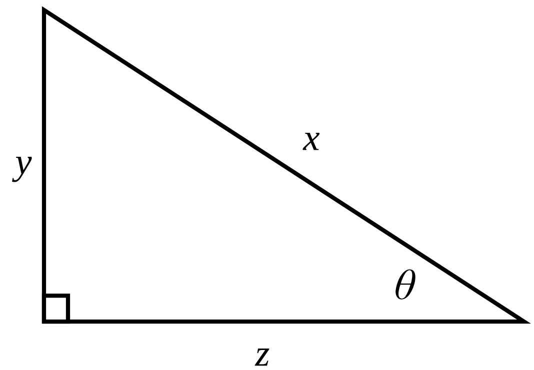
When right triangles (or any triangles for that matter) are labeled in the way depicted in the figure, the following rules apply.

- Angles are labeled with upper case letters; A, B, C, etc.
- Sides are labeled with lower case letters; *a*, *b*, *c*, etc.
- The triangle is set up in a manner such that every lower case letter marks the side opposite the angle labeled with the corresponding upper case letter;

  In other words; side *a* is opposite angle A, side *b* is opposite angle B, etc.

Right triangles are sometimes labeled less explicitly as shown in the figure below.

Figure 2



When labeling a triangle in this way, you will need to know the following:

- ***θ*** (the Greek letter theta) is the variable that is the most widely used to label an unknown angle.
- The ***hypotenuse*** is the side that is opposite the right angle.
- The two sides that make up the right angle are called ***legs***.
- The legs are designated as ***opposite*** (which means it is opposite θ, or the angle in question) and ***adjacent*** (which means it runs along the side of θ, or the angle in question).

Given this information answer the questions below:

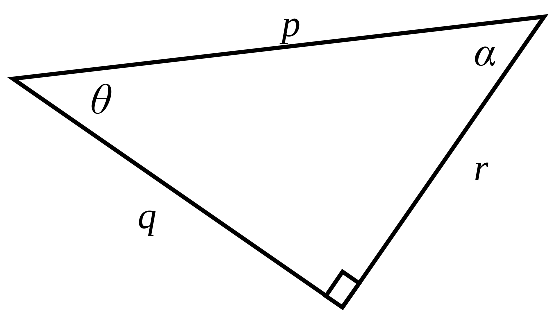
1. Which side is the hypotenuse?

2. Which side is the leg opposite θ?

3. Which side is the leg adjacent to θ?

Given the triangle below:

Figure 3



Fill in the following table with the appropriate side.

	ANGLE	
	θ	**α**
OPPOSITE		
ADJACENT		
HYPOTENUSE		

THE PYTHAGOREAN THEOREM

Most students know the Pythagorean Theorem but they sometimes over-generalize 1 formula for it. Often times, students are given a right triangle that is marked as it is in Figure 1. Given such a triangle, then the Pythagorean Theorem would lead to the following equation.

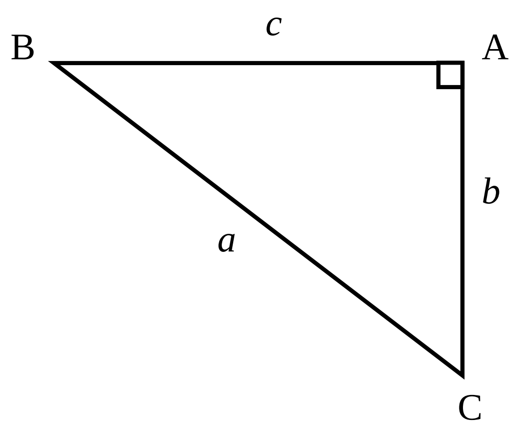
$$a^2 + b^2 = c^2$$

However, this is the formula for a right triangle labeled in the SPECIFIC way it was labeled in Figure 1. Students should be aware of the actual Pythagorean Theorem.

Pythagorean Theorem: *For any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs.*

4. Given this information, write the equation that would be given by the Pythagorean Theorem for the triangle given below.

Figure 4



5. Write the equation that would be given by the Pythagorean theorem for the right triangle indicated:

(A) Figure 2

(B) Figure 3

As you progress through this lab, as well as this course in Trigonometry, be aware of the things discussed here as a slight misunderstanding in these areas can lead to quite a bit of confusion. A couple seconds of thought can keep you from having major complications in Trigonometry.

PART 2: STUDYING RIGHT TRIANGLES

1. Open the RIGHT TRIANGLE webpage. This applet allows you to manipulate a right triangle and then provides you the approximate measurement of the sides and angles (measure up to 2 decimal places). As with any lab investigation there will always be some error in measurement.

2. Move points A and B to create a right triangle of any size and shape. Make sure that your angles are as close to the nearest whole-numbered measurement as possible.

3. Fill in Table 1 as directed below.

(A) Round the angles to the nearest whole number. [Again make sure that you have angles that are as close to the nearest whole-numbered angle as possible.]

(B) Record the sides of your triangle in the row marked, "Triangle 1." Use the number of decimal places that the applet gives you, generally this will be 2.

(C) Change the triangle so that the side measurements are different. The angle measurements of this new triangle should be roughly the same. [Again you will want to get as close as you can to the nearest whole-numbered angle, but you may not be able to get EXACTLY the same angles you for Triangle 1.] Enter these side measurements in the row marked, "Triangle 2."

(D) Repeat step C above to make a third right triangle that has noticeably different side measurements as Triangle 1 and Triangle 2, but has angles roughly equal to those seen in those previous triangles. Enter the side measurements you obtained in the row marked, "Triangle 3."

(E) Use your calculator to carry out the calculations requested in the last 8 columns of the table. Round these calculations you make to 1 decimal place.

TABLE 1

$\angle A =$ ______ $\angle B =$ ______

	a	b	c	$a^2 + b^2$	c^2	a/b	a/c	b/c	b/a	c/a	c/b
TRIANGLE 1											
TRIANGLE 2											
TRIANGLE 3											

4. Repeat the procedure in step 3 to create a different family of triangles and enter the data below in Table 2. That is, the triangles below should have angles A and B be noticeably different from those in Table 1.

TABLE 2

$\angle A =$ ______ $\angle B =$ ______

	a	b	c	$a^2 + b^2$	c^2	a/b	a/c	b/c	b/a	c/a	c/b
TRIANGLE 1											
TRIANGLE 2											
TRIANGLE 3											

Use the two tables you completed here to complete parts 3 and 4.

PART 3: INITIAL FINDINGS

1. According to your findings in Table 1 and Table 2, is the Pythagorean Theorem true? What information from the table confirms this?

2. In Table 1, are any of the values for Triangle 1, Triangle 2, and Triangle 3 the same (or approximately the same due to the rounding you were asked to do)? If so, which ones?

3. Do you find the same phenomenon that you described in #2 above occurring in the triangles you used for Table 2?

4. What GEOMETRIC phenomenon explains what you found in #2 and #3?

PART 4: DEFINING SINE, COSINE & TANGENT

❈ Before continuing, make sure that your calculator is set to DEGREE mode.

(A) You will now be using information from Table 1 (from Part 2) and complete some calculations in order to complete Table 3 on the next page. Follow the steps listed in order to complete Table 3 correctly.

1. Fill in the values of angles A and B on the table where indicated.

2. Fill in the value for $\sin\angle A$, $\cos\angle A$, $\tan\angle A$, $\sin\angle B$, $\cos\angle B$, and $\tan\angle B$ by calculating the values on your calculator. Round these values to 1 decimal place. These values should appear under the "VALUE" column of Table 3 (next page).

3. Each of the values you obtained in #2 correspond (or at least they should) to a value of one of the last 6 columns from Table 1. Please ignore any insignificant differences as there was rounding involved in several areas of the lab thus far. Fill in the column heading from Table 1 that has the same value as the trigonometric function given in Table 3. The first one has been done for you.

4. Answer the following questions about the triangle you used for Table 1 in Part 2. [Note: the following would be true of ANY right triangle you construct with the web page.]

(a) Which side is the leg opposite of angle A?

(b) Which side is the leg adjacent to angle A?

(c) Which side is the leg opposite of angle B?

(d) Which side is the leg adjacent to angle B?

(e) Which side is the hypotenuse?

5. Use the information in this section to write the "Definition Formulas" for the Trigonometric functions. You should use the terms ***opposite***, ***adjacent*** and ***hypotenuse***. The first one has been done for you.

TABLE 3

$\angle A =$ ________ $\angle B =$ ________

TRIGONOMETRIC FUNCTION	VALUE	FIGURE FORMULA	DEFINITION FORMULA
sin (A)		a/c	$\frac{\text{opposite}}{\text{hypotenuse}}$
cos (A)			
tan (A)			
sin (B)			
cos (B)			
tan (B)			

6. Repeat steps 1-5, but now use the information you gathered in Table 2 to fill in Table 4 below.

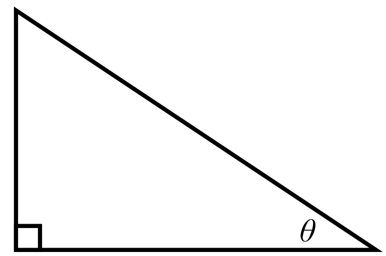
TABLE 4

$\angle A =$ ________ $\angle B =$ ________

TRIGONOMETRIC FUNCTION	VALUE	FIGURE FORMULA	DEFINITION FORMULA
sin (A)		a/c	$\frac{\text{opposite}}{\text{hypotenuse}}$
cos (A)			
tan (A)			
sin (B)			
cos (B)			
tan (B)			

7. Based on your findings, are the definition formulas for sine, cosine and tangent the same regardless of the angle used? Explain your response.

8. Based on your findings in part 4 and given the figure to the right…



1. Label the opposite leg, adjacent leg and hypotenuse on the triangle using θ as the angle of interest.

2. Write the definition of the three trigonometric functions below.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

<u>PART 5</u>: SPECIAL RIGHT TRIANGLES AND TRIGONOMETRIC FUNCTIONS

In this section we will be focusing on two special right triangles. One is the 30-60-90 right triangle and the other is the isosceles right triangle (also known as the 45-45-90 triangle). When we deal with these types of right triangles some specific values are mentioned a great deal. Precalculus textbooks use exact values. Here we have been using a right triangle on a web page. The sides measured on the triangles you create in the web page are rounded.

We will be looking to find EXACT values here too, but since they involve irrational numbers, the calculator alone will not be able to do this for us. So you will need to be aware of the decimal read-outs you get on the calculator when dealing with these values.

1. Calculate the following on your calculator. Write the answer you get for each with <u>2</u> decimal places.

   $\sqrt{2} =$ _______ $\qquad \frac{\sqrt{2}}{2} =$ _______ $\qquad \frac{1}{\sqrt{2}} =$ _______

   Which of the above values are equal to one another? What can you conclude from this? [Write your final response as an equation.]

2. Calculate the following on your calculator. Write the answer you get for each with <u>2</u> decimal places.

   $\sqrt{3} =$ _______ $\qquad \frac{\sqrt{3}}{3} =$ _______ $\qquad \frac{\sqrt{3}}{2} =$ _______ $\qquad \frac{1}{\sqrt{3}} =$ _______

   Which of the above values are equal to one another? What can you conclude from this? [Write your final response as an equation.]

Now we will look at these two special right triangles and explore the sine, cosine, and tangent of angles that are associated with these triangles.

30-60-90 TRIANGLE

Use the RIGHT TRIANGLE web page to create any 30-60-90 triangle (that is a right triangle with the other two angles being 30º and 60º. As in the previous sections you may have to get as close as you can to these values as possible rather than exactly.)

Now that you have a 30-60-90 triangle, fill in the information below.

$\angle$A= __________ $\angle$B= __________

a = __________ b = __________ c = __________

1. Use the sides of your triangle to calculate the following values. Show your work to demonstrate that you are calculating them as directed. Round your answers to 2 decimal places.

$\sin\left(30^o\right) =$ $\sin\left(60^o\right) =$

$\cos\left(30^o\right) =$ $\cos\left(60^o\right) =$

$\tan\left(30^o\right) =$ $\tan\left(60^o\right) =$

2. Use your calculator to find the six trigonometric functions listed above. Do your results above, in general, agree with your calculator's output?

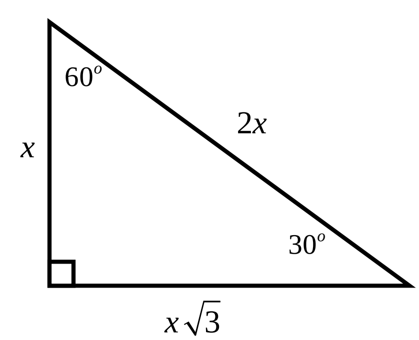
3. Now, use the EXACT values you explored on page 10, as well as common sense, to write the EXACT values of the these trigonometric functions below.

$\sin\left(30^o\right) =$ $\sin\left(60^o\right) =$

$\cos\left(30^o\right) =$ $\cos\left(60^o\right) =$

$\tan\left(30^o\right) =$ $\tan\left(60^o\right) =$

4. On the right is a diagram that is often used to as the diagram for a general 30-60-90 triangle that is found in textbooks and even the SAT. Do the results you obtained here verify that this diagram is accurate? Explain your answer.



ISOSCELES RIGHT (45-45-90) TRIANGLE

Use the RIGHT TRIANGLE web page to create any isosceles right triangle. (To do this, make sure that the two legs have equal length. The angles may not both be 45 degrees exactly, but again there is some error of measurement in the applet.)

Now that you have an isosceles right triangle, fill in the information below.

$\angle A =$ __________ $\angle B =$ __________

$a =$ __________ $b =$ __________ $c =$ __________

1. **1.** Use the sides of your triangle to calculate the following values. Show your work to demonstrate that you are calculating them as directed. Round your answers to 2 decimal places.

$$\sin(45^o) =$$

$$\cos(45^o) =$$

$$\tan(45^o) =$$

2. **2.** Use your calculator to find the six trigonometric functions listed above. Do your results above, in general, agree with your calculator's output?

3. **3.** Now, use the EXACT values you explored on page 10, as well as common sense, to write the EXACT values of the these trigonometric functions below.

$$\sin(45^o) =$$

$$\cos(45^o) =$$

$$\tan(45^o) =$$

4. **4.** On the right is a diagram that is often used to as the diagram for a general isosceles right triangle that is found in textbooks and even the SAT. Do the results you obtained here verify that this diagram is accurate? Explain your answer.