TRIGONOMETRIC FUNCTION GRAPHS

The graphs of the trigonometric functions are rather unique in Mathematics. Because of how they are formed these graphs are **periodic**, that is, they will exhibit a repeating pattern. Several things determine how this repeating pattern will develop. For now though, we are simply going to explore the graphs of these functions.

We have already worked with these functions and are aware of their origins and the values of these trigonometric functions for specific angle measurements in both degrees and radians. For the purposes of this exploration, we will focus on using angles in radian measurement only. As you progress through this activity, you should make use of the values for these trigonometric functions as you should have them memorized by this time. The only difference is here is that you will see how those values demonstrate the appearances of the graphs for these functions.

You will be making use of your calculator for this activity as well. By this time you should have already been working with radians and, therefore, your calculator should be in radian mode. Make sure that this is the case throughout your work with this activity.

PART 1: $y = \sin\theta$

(A) Use your knowledge of trigonometric functions and your calculator to fill in the following table. Round the *y*-values to three decimal places.

(B) Replacing θ with *x*, use the table you created in part (A) to graph $y = \sin x$. Complete the graph based on the pattern you see. Confirm your graph by graphing the function on your calculator using the window below.

θ	$y = \sin\theta$ EXACT	$y = \sin\theta$ DECIMAL
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		
$7\pi/6$		
$5\pi/4$		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
$7\pi/4$		
$11\pi/6$		
2π		
$13\pi/6$		
$9\pi/4$		
$7\pi/3$		
$5\pi/2$		

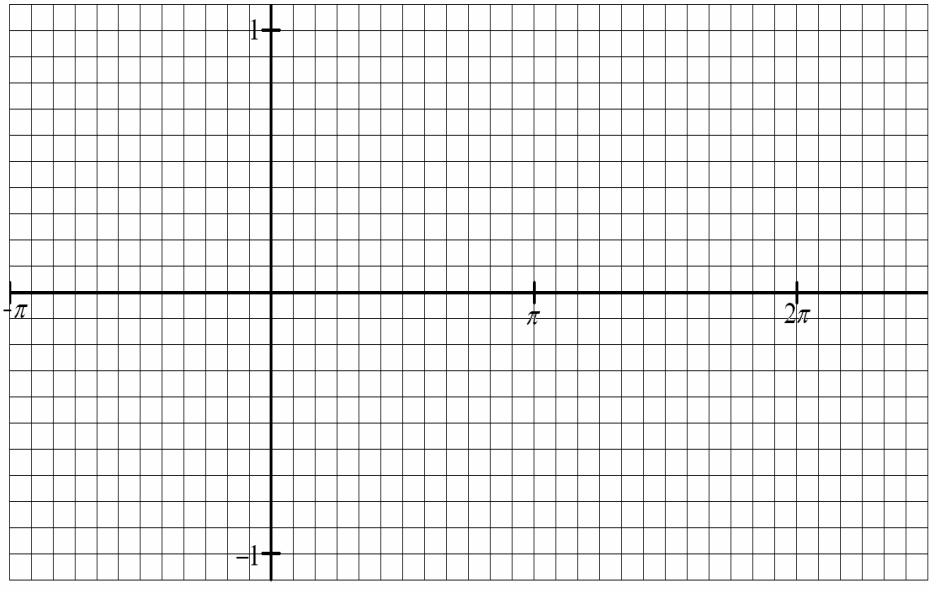
```
WINDOW
 Xmin=-3.141592...
 Xmax=7.8539816...
 Xscl=.26179938...
 Ymin=-1.1
 Ymax=1.1
 Yscl=.1
 Xres=1
```

Note: If you enter the following values for Xmin, Xmax, and Xscl, you will obtain the values seen above.

$$\text{Xmin} = -\pi$$

$$\text{Xmax} = 5\pi / 2$$

$$\text{Xscl} = \pi / 12$$



Now let's look at the graph we just produced. Here I will point out some features of the graph that will be important later as we do more with these functions. As you progress through this worksheet packet, you will become more responsible for obtaining this information.

DOMAIN:

- Are there any domain restrictions for $y = \sin\theta$?
- What is the domain of $y = \sin\theta$?

RANGE:

Based on what you see in the graph of $y = \sin\theta$**,** what is the range of the function?

PERIOD:

This is a feature of graphs that was mentioned before, but we really have not worked with it yet. Trigonometric function graphs have a repeating pattern, in that a general shape is repeated over and over again. Upon studying the graph you produced for $y = \sin\theta$, what is the period of this function?

***y*-INTERCEPT:**

Based on the graph you produced, what are the coordinates of the *y*-intercept for $y = \sin\theta$?

BASELINE:

The baseline of a periodic graph is a line that essentially cuts the graph in half. For $y = \sin\theta$ what is the equation of the baseline? [Hint: For this function the baseline is horizontal.]

AMPLITUDE:

The amplitude is basically the furthest distance that a periodic graph is from the baseline. Based on this description, what is the amplitude of $y = \sin\theta$?

Now that all of the information has been provided to you at this time, it's your turn to recount this information.

FUNCTION	$y = \sin\theta$
DOMAIN	
RANGE	
PERIOD	
y**-INTERCEPT**	
BASELINE	
AMPLITUDE	

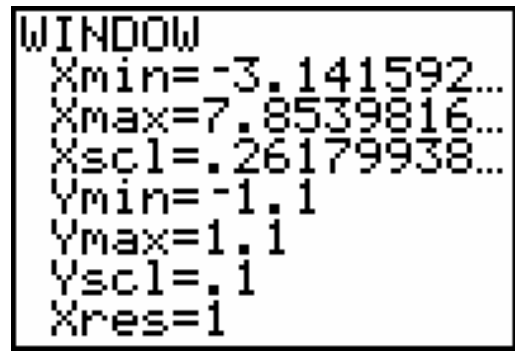
Now explore the next Trigonometric Function.

PART 2: $y = \cos\theta$

(A) Use your knowledge of trigonometric functions and your calculator to fill in the following table. Round the *y*-values to three decimal places.

(B) Replacing θ with *x*, use the table you created in part (A) to graph $y = \cos x$. Complete the graph based on the pattern you see. Confirm your graph by graphing the function on your calculator using the window below.

θ	$y = \cos\theta$ EXACT	$y = \cos\theta$ DECIMAL
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		
$7\pi/6$		
$5\pi/4$		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
$7\pi/4$		
$11\pi/6$		
2π		
$13\pi/6$		
$9\pi/4$		
$7\pi/3$		
$5\pi/2$		

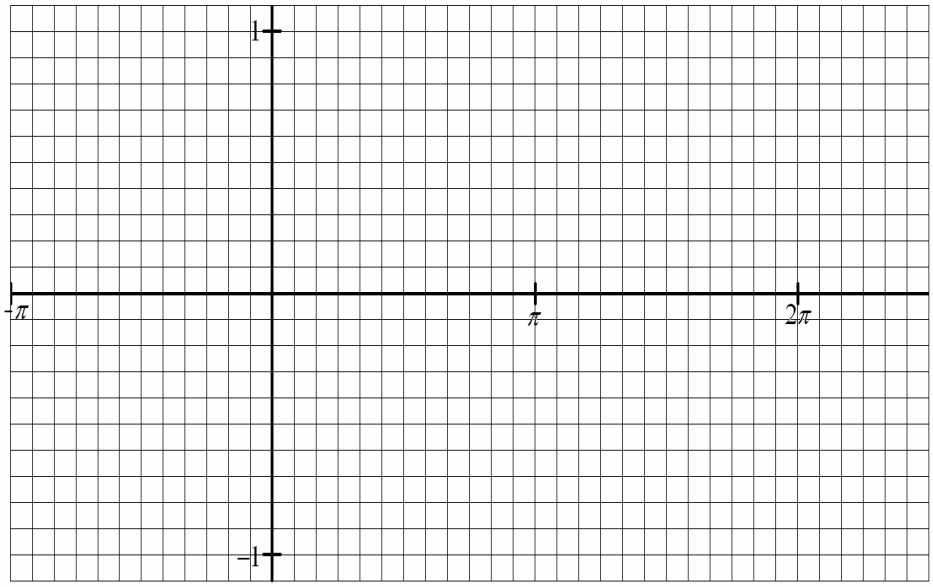


Note: If you enter the following values for Xmin, Xmax, and Xscl, you will obtain the values seen above.

$$\text{Xmin} = -\pi$$

$$\text{Xmax} = 5\pi/2$$

$$\text{Xscl} = \pi/12$$



OBSERVATIONS:

Fill in the table below. Use your table of $y = \cos\theta$ and the accompanying graph, as well as, the summary information from part 1 to guide you through this process.

FUNCTION	$y = \cos\theta$
DOMAIN	
RANGE	
PERIOD	
y**-INTERCEPT**	
BASELINE	
AMPLITUDE	

What are the similarities between $y = \cos\theta$ and $y = \sin\theta$?

What are the differences between $y = \cos\theta$ and $y = \sin\theta$?

As you can see, there are a lot of similarities between the graphs of $y = \cos\theta$ and $y = \sin\theta$. Their shapes are identical with the only exception that they are orientated on the coordinate system differently. That is why the shape seen in both of these graphs is called a **SINE FUNCTION**.

The remaining trigonometric functions however, do not have this shape. They will still be periodic, but they will start having other features that are different from these other two functions.

One thing that will be noticeably different is that these graphs will have **vertical asymptotes.**

VERTICAL ASYMPTOTES

Vertical asymptotes occur where functions obtain undefined values. Let's look at the function $y = \tan\theta$ as a test case.

Use your calculator to find the value of $\tan\left(\frac{\pi}{2}\right)$. What happened? Well, let's do away with the suspense…you will get the following.

- According to your knowledge of the table of common trigonometric values that you should have memorized by now, what is the value of $\tan\left(\frac{\pi}{2}\right)$?

- Therefore, if you are using the calculator to evaluate a trigonometric function for a particular angle and you get the error message described earlier, what does that mean?

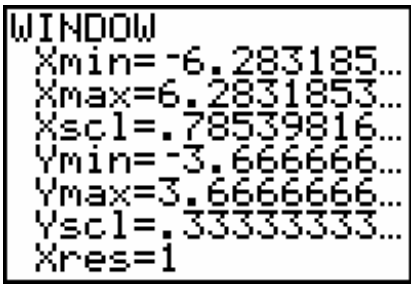
The following is a graph calculator-generated graph of this function and the window in which the graph was created.

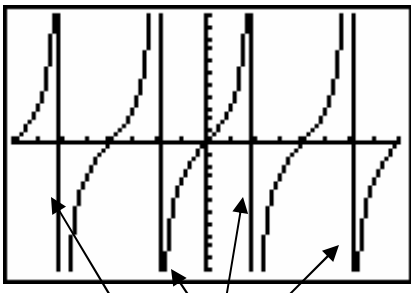
Note: If you enter the following values for Xmin, Xmax, and Xscl, you will obtain the values seen to the right.

$\text{Xmin} = -2\pi$

$\text{Xmax} = 2\pi$

$\text{Xscl} = \pi / 4$





VERTICAL ASYMPTOTES

As you can see, the graph of this function produces vertical asymptotes. The calculator shows them as vertical lines, however, you should be aware that technically, these lines should not be seen. However, seeing them allows you to have an idea where these asymptotes occur.

- Based on the picture of the graph produced on your calculator (and the one shown on the previous page), where do these vertical asymptotes appear to occur?

- What happens when you attempt to take the tangent of those angles on your calculator?

- Based on this, what does a vertical asymptote indicate in regards to the tangent function?

[Note: Vertical asymptotes have the same meaning in any function in which they occur.]

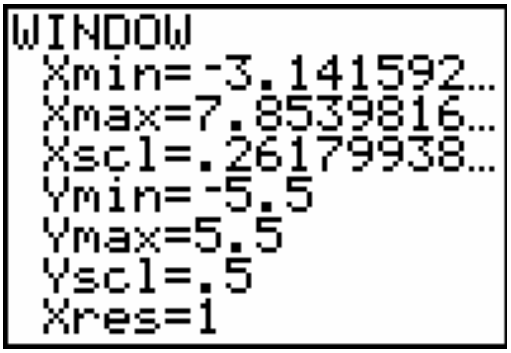
Now that we have had a general look at the tangent function, let's look at it in a bit more detail…

PART 3: $y = \tan\theta$

(A) Use the information from the previous page, your knowledge of trigonometric functions and your calculator to fill in the following table. Round the *y*-values to three decimal places.

(B) Replacing θ with *x*, use the table you created in part (A) to graph $y = \tan x$. Complete the graph based on the pattern you see. Confirm your graph by graphing the function on your calculator using the window below.

θ	$y = \tan\theta$ EXACT	$y = \tan\theta$ DECIMAL
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		
$7\pi/6$		
$5\pi/4$		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
$7\pi/4$		
$11\pi/6$		
2π		
$13\pi/6$		
$9\pi/4$		
$7\pi/3$		
$5\pi/2$		

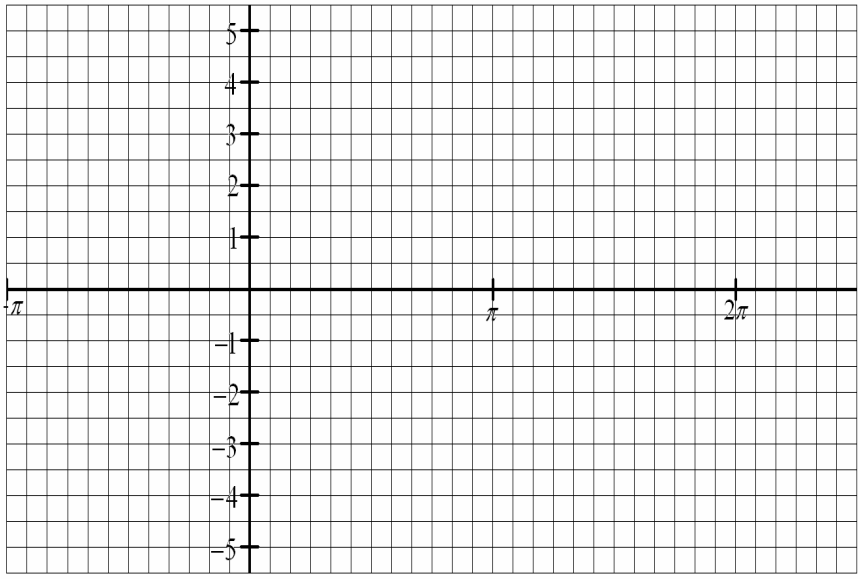


Note: If you enter the following values for xMin, xMax, and xScl, you will obtain the values seen above.

$$\text{Xmin} = -\pi$$

$$\text{Xmax} = 5\pi/2$$

$$\text{Xscl} = \pi/12$$



OBSERVATIONS:

DOMAIN:

We have already discussed the issues with $\tan\left(\frac{\pi}{2}\right)$. However, as seen on your table and in the graph you drew and that you viewed on your calculator $\frac{\pi}{2}$ is not the only value that produces undefined values for $y = \tan\theta$.

- What are the other values of θ on your table in which the same thing occurs?

- In interval notation, the domain of $y = \tan\theta$ is typically expressed as all real numbers except odd multiples of $\frac{\pi}{2}$.

  In interval notation this is $x \in (-\infty, \infty) \cap \left\{ x \neq \frac{(2n+1)\pi}{2}; n \in \mathbb{Z} \right\}$.

  Do your observations agree with this assertion? Explain.

RANGE:

Based on what you see in your graph, what is the range of $y = \tan\theta$**?**

PERIOD:

This graph is still periodic. In fact, because of the places where the graph has breaks caused by the vertical asymptotes, it is easy to see where the graph repeats. So what is the period of $y = \tan\theta$?

***y*-INTERCEPT:**

What is the *y*-intercept of $y = \tan\theta$?

BASELINE:

What is the equation of the baseline of $y = \tan\theta$?

AMPLITUDE:

- Does the graph of $y = \tan\theta$ have any point that lies on it that you can describe as a highest or lowest point?

- Based on your answer to the last question, what is the amplitude of $y = \tan\theta$?

Now given all of this information fill in the table below

FUNCTION	$y = \tan\theta$
DOMAIN	
RANGE	
PERIOD	
y **-INTERCEPT**	
BASELINE	
AMPLITUDE	

THE INVERSE TRIGONOMETRIC FUNCTIONS

You have been using the calculator to check your work on the graphs of these functions but the last three functions are not readily available on the calculator. So to enter these in your calculator you have to be familiar with the definition of these functions and enter the following.

$$y = \csc\theta \rightarrow y = 1/\sin(x)$$
$$y = \sec\theta \rightarrow y = 1/\cos(x)$$
$$y = \cot\theta \rightarrow y = 1/\tan(x)$$

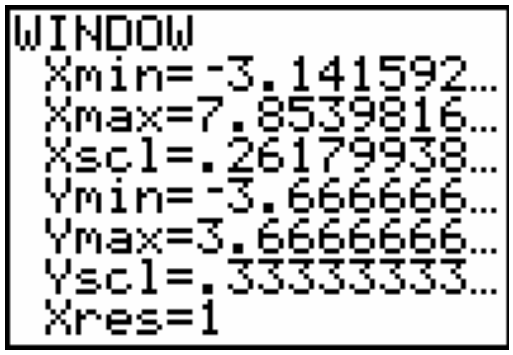
Now you have enough background to complete the last three parts of this exploration.

PART 4: $y = \csc\theta$

(A) Use your knowledge of trigonometric functions and your calculator to fill in the following table. Round the *y*-values to three decimal places.

(B) Replacing θ with *x*, use the table you created in part (A) to graph $y = \csc x$. Complete the graph based on the pattern you see. Confirm your graph by graphing the function on your calculator using the window below.

θ	$y = \csc\theta$ EXACT	$y = \csc\theta$ DECIMAL
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		
$7\pi/6$		
$5\pi/4$		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
$7\pi/4$		
$11\pi/6$		
2π		
$13\pi/6$		
$9\pi/4$		
$7\pi/3$		
$5\pi/2$		

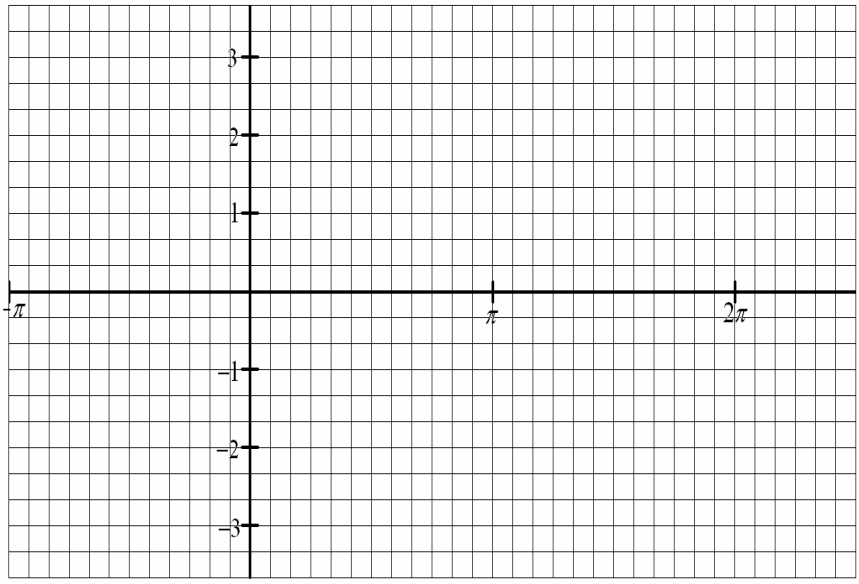


Note: If you enter the following values for xMin, xMax, and xScl, you will obtain the values seen above.

$$\text{Xmin} = -\pi$$

$$\text{Xmax} = 5\pi / 2$$

$$\text{Xscl} = \pi / 12$$



SUMMARY INFORMATION

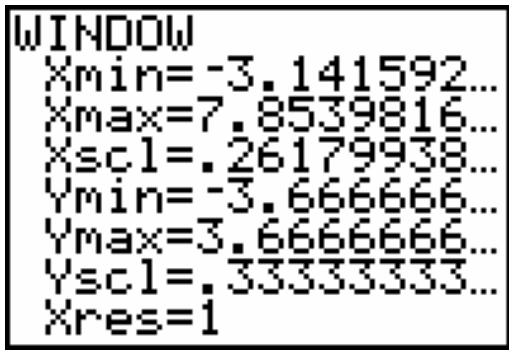
FUNCTION	$y = \csc\theta$
DOMAIN	
RANGE	
PERIOD	
y**-INTERCEPT**	
BASELINE	
AMPLITUDE	

PART 5: $y = \sec\theta$

(A) Use your knowledge of trigonometric functions and your calculator to fill in the following table. Round the *y*-values to three decimal places.

(B) Replacing θ with *x*, use the table you created in part (A) to graph $y = \sec x$. Complete the graph based on the pattern you see. Confirm your graph by graphing the function on your calculator using the window below..

θ	y = sec θ EXACT	y = sec θ DECIMAL
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		
$7\pi/6$		
$5\pi/4$		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
$7\pi/4$		
$11\pi/6$		
2π		
$13\pi/6$		
$9\pi/4$		
$7\pi/3$		
$5\pi/2$		

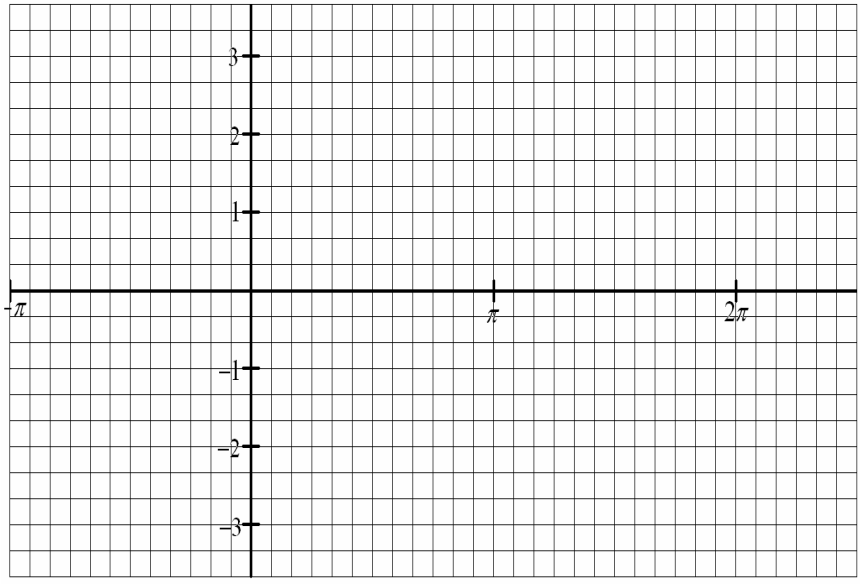


Note: If you enter the following values for xMin, xMax, and xScl, you will obtain the values seen above.

$$\text{Xmin} = -\pi$$

$$\text{Xmax} = 5\pi / 2$$

$$\text{Xscl} = \pi / 12$$



SUMMARY INFORMATION

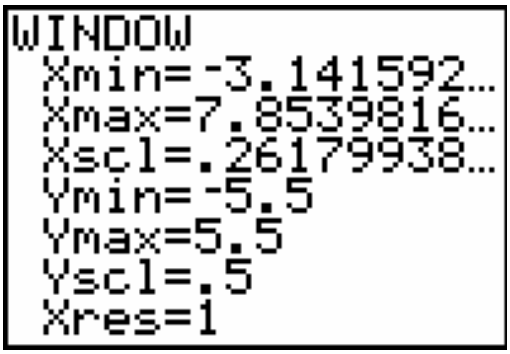
FUNCTION	$y = \sec\theta$
DOMAIN	
RANGE	
PERIOD	
y**-INTERCEPT**	
BASELINE	
AMPLITUDE	

PART 6: $y = \cot\theta$

(A) Use your knowledge of trigonometric functions and your calculator to fill in the following table. Round the *y*-values to three decimal places.

(B) Replacing θ with *x*, use the table you created in part (A) to graph $y = \cot x$. Complete the graph based on the pattern you see. Confirm your graph by graphing the function on your calculator using the window below.

θ	$y = \cot\theta$ EXACT	$y = \cot\theta$ DECIMAL
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		
$7\pi/6$		
$5\pi/4$		
$4\pi/3$		
$3\pi/2$		
$5\pi/3$		
$7\pi/4$		
$11\pi/6$		
2π		
$13\pi/6$		
$9\pi/4$		
$7\pi/3$		
$5\pi/2$		

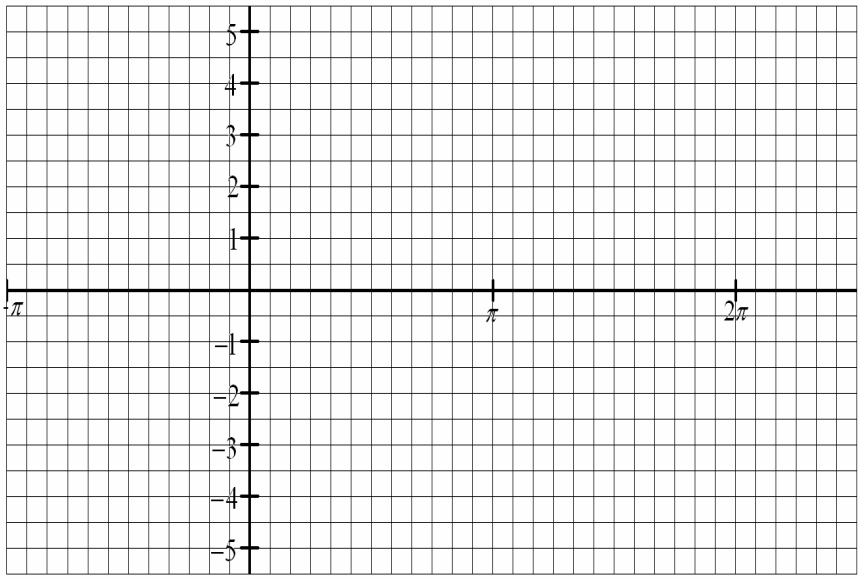


Note: If you enter the following values for xMin, xMax, and xScl, you will obtain the values seen above.

$$\text{Xmin} = -\pi$$

$$\text{Xmax} = 5\pi / 2$$

$$\text{Xscl} = \pi / 12$$



SUMMARY INFORMATION

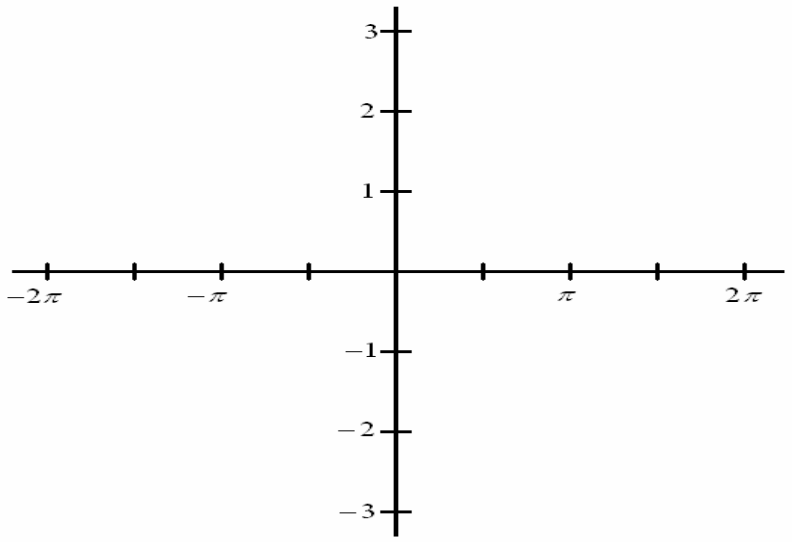
FUNCTION	$y = \cot\theta$
DOMAIN	
RANGE	
PERIOD	
y**-INTERCEPT**	
BASELINE	
AMPLITUDE	

FINAL SUMMARY: TRIGONOMETRIC FUNCTIONS

Fill in the following information.

FUNCTION: $y = \sin\theta$

SKETCH



DOMAIN:

RANGE:

***y*-INTERCEPT:**

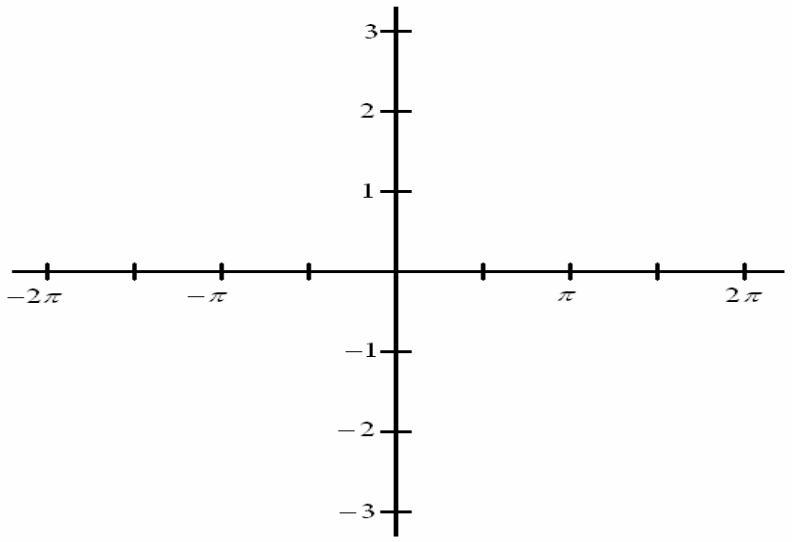
BASELINE:

PERIOD:

AMPLITUDE:

FUNCTION: $y = \cos\theta$

SKETCH



DOMAIN:

RANGE:

***y*-INTERCEPT:**

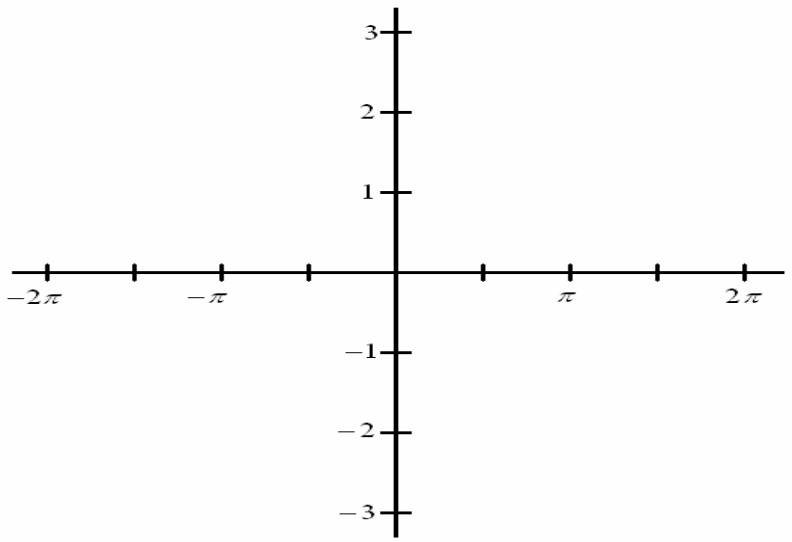
BASELINE:

PERIOD:

AMPLITUDE:

FUNCTION: $y = \tan\theta$

SKETCH



DOMAIN:

RANGE:

***y*-INTERCEPT:**

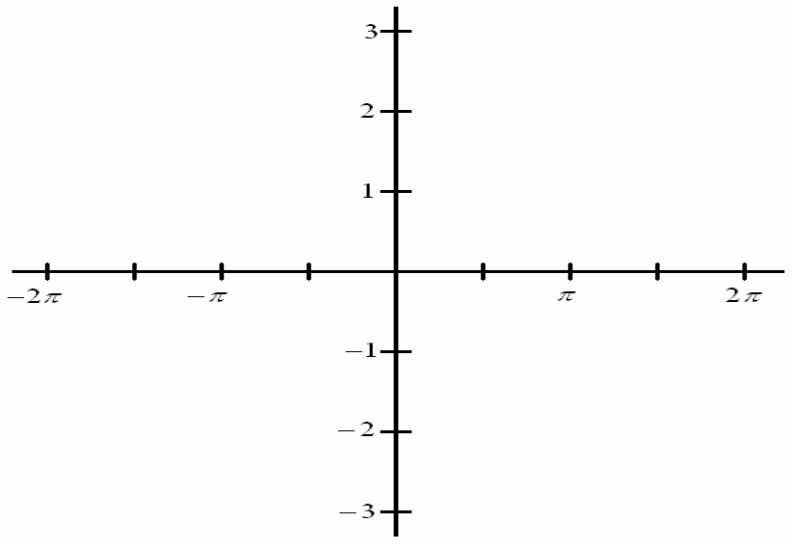
BASELINE:

PERIOD:

AMPLITUDE:

FUNCTION: $y = \csc\theta$

SKETCH



DOMAIN:

RANGE:

***y*-INTERCEPT:**

BASELINE:

PERIOD:

AMPLITUDE:

FUNCTION: $y = \sec\theta$

SKETCH

DOMAIN:

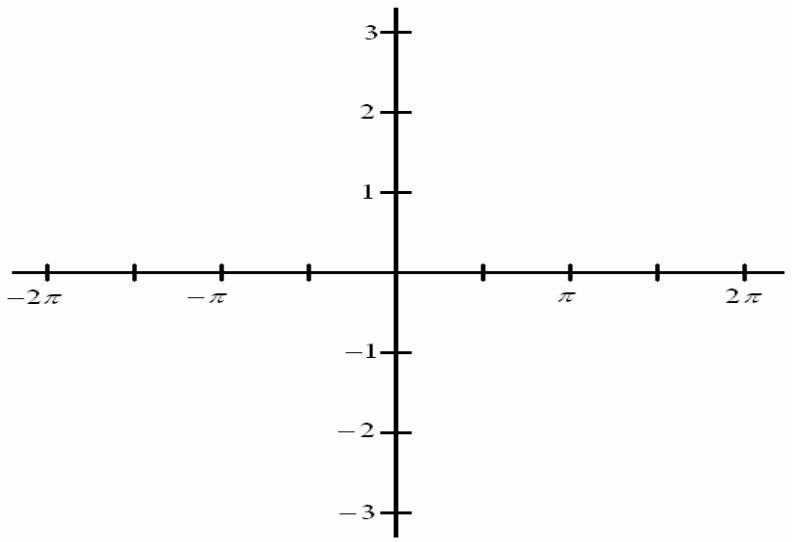
RANGE:

***y*-INTERCEPT:**

BASELINE:

PERIOD:

AMPLITUDE:



FUNCTION: $y = \cot\theta$

SKETCH

DOMAIN:

RANGE:

***y*-INTERCEPT:**

BASELINE:

PERIOD:

AMPLITUDE: