

SEQUENCES INVESTIGATION

A sequence is a list of numbers that, in most cases, have a specific pattern to them. In this investigation we will be exploring two types of sequences; ARITHMETIC and GEOMETRIC.

For this investigation, we will be using an Excel workbook that I have set up for you entitled REGRESSION CALCULATOR & GRAPHER. This is the file REGRESSIONS found in the EXCEL-WKSHT folder. [See the DOWNLOADING EXCEL-WKSHT document for instructions in obtaining this file.]

This can also be done using your graphing calculator or Graph 4.3 (see my webpage for details on this program), however, different steps that what are listed here would need to be done using them.

ARITHMETIC SEQUENCES:

- (1) An example of an arithmetic sequence is shown below...

5, 8, 11, 14, 17, 20, 23, 26, 29,...

The reason the sequence is an arithmetic sequence is that as you move from one term to the next, the change in value is caused by adding a constant value (called the **common difference**).

What is the common difference for this sequence?

- (2) Now complete the table below. n represents the position of the term (i.e. $n = 1$ means the 1st term, $n = 2$ is the 2nd term, etc.) and u_n represents the value of the term in that position (i.e. $u_3 = 11$ because 11 is the 3rd term in the sequence).

n	u_n
1	5
2	8
3	11
4	
5	
6	
7	
8	
9	
10	
11	
12	

- (3) Open the Excel file REGRESSION CALCULATOR & GRAPHER, and click on the tab marked POLY-EXP-LOG. Enter in the table you completed in step #2 [Let $x = n$ and let $y = u_n$]

(A) What shape is the pattern made by the data points?

(B) Which regression listed on the Excel worksheet should you use to find the equation that best fits that pattern?

- (4) Run the appropriate regression. How well does the regression line/curve fit the data points? (Look both at the graph and to see how close R^2 is to 1.)

- (5) What is the equation given by the regression function? (Look above R^2 in the yellow box.)

- (6) Take the equation from #5 and replace x with n and replace y with u_n . Write the equation here.

- (7) Do you see the value for the constant difference anywhere in the equations you found in #5 and #6? What feature of the line is represented by the common difference of the sequence?

- (8) The formula you found in #6 is called an **explicit formula** because substituting the terms position in the sequence will compute the value of that term (without having to write out all prior terms). Based on this, what is the 100th term of this sequence?

- (9) Repeat the steps we did in #1-6 to complete the table below and write the explicit formula for the sequence:

100, 96, 92, 88,...

n	u_n
1	100
2	96
3	92
4	88
5	
6	
7	
8	
9	
10	
11	
12	

- (10) Based on #9, can the common difference be negative?
- (11) What would the 55th term of the sequence from #9 be?
- (12) According to your textbook the explicit formula for an arithmetic sequence is given by:

$$u_n = u_1 + (n - 1) \cdot d$$

n = the position of the term

u_n = the value of the term in position n

u_1 = 1st term

d = the common difference

Based on this...

- (A) What is the explicit formula of the sequence from #1 according to the book's formula above?
- (B) What is the explicit formula of the sequence from #9 according to the book's formula above?
- (C) Simplify the formulas from (A) and (B) [substitute for u_1 and d and then simplify the expression]. Do these agree with the explicit formulas you came up with in #6 and #9?

GEOMETRIC SEQUENCES:

- (1) An example of an geometric sequence is shown below...

8, 12, 18, 27,...

The reason the sequence is a geometric sequence is that as you move from one term to the next, the change in value is caused by multiplying a constant value (called the **common ratio**).

What is the common ratio for this sequence?

- (2) Now complete the table below. n represents the position of the term (i.e. $n = 1$ means the 1st term, $n = 2$ is the 2nd term, etc.) and u_n represents the value of the term in that position (i.e. $u_3 = 18$ because 18 is the 3rd term in the sequence).

n	u_n
1	8
2	12
3	18
4	27
5	
6	
7	
8	
9	
10	
11	
12	

- (3) Open the Excel file REGRESSION CALCULATOR & GRAPHER, and click on the tab marked POLY-EXP-LOG. Enter in the table you completed in step #2 [Let $x = n$ and let $y = u_n$]
- (A) The shape of the curve made of the data points appears to be what type of function?
- (B) Which regression listed on the Excel worksheet should you use to find the equation that best fits that pattern? [This may require some experimentation to determine...again look at both the fit of the curve to the data points and how close R^2 is to 1.]
- (4) What is the equation given by the regression function? (Look above R^2 in the yellow box.)
- (5) Take the equation from #4 and replace x with n and replace y with u_n . Write the equation here. Again this will be the explicit formula for this sequence.
- (6) Based on your answer for #5, what is the 25th term of this sequence?

- (7) Repeat the steps we did in #1-5 to complete the table below and write the explicit formula for the sequence:

999, 333, 111, 37,...

n	u_n
1	999
2	333
3	111
4	37
5	
6	
7	
8	
9	
10	
11	
12	

Common Ratio = _____

- (8) What would the 50th term of the sequence from #7 be?
- (9) According to your textbook the explicit formula for an geometric sequence is given by:

$$u_n = u_1 \cdot r^{n-1}$$

n = the position of the term

u_n = the value of the term in position n

u_1 = 1st term

r = the common ratio

Based on this...

- (A) What is the explicit formula of the sequence from #1 according to the book's formula above?
- (B) What is the explicit formula of the sequence from #7 according to the book's formula above?

(C) Do the formulas you found in parts (A) and (B) produce the same results that we found using Excel? Explain and show any pertinent work.

(D) Which formula is better to use, the one Excel produces or the one that the book presents? Explain your answer.

EXERCISES

Use what you learned in this investigation to...

(A) write the explicit formula of the sequence

(B) find the 20th term of the sequence

1. 0.25, 1, 4, 16,...

2. 26, 35, 44, 53,...

3. 88, 75, 62, 49,...

4. 200, 20, 2, 0.2,...

5. 8, 10.5, 13, 15.5,...

6. 8, 10.8, 14.58, 19.683,...

7. $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

8. 6.54, 6.31, 6.08, 5.85,...

9. 5, 6, 7.2, 8.64,...

10. $\frac{3}{10}, \frac{3}{2}, \frac{15}{2}, \frac{75}{2}$