MATH LAB: TRIGONOMETRY OF NON-RIGHT TRIANGLES

In order to complete parts 1 & 2 you will need to use the applet found on the web page PYTHAGOREAN SQUARES. The link to this webpage is given below:

http://atsorren.freewebpages.org/GENERAL/PYTHSQUARE/pythsqr.html

PART 1: THE PYTHAGOREAN THEOREM

1. Open the web page and set angle C to 90° by using the slider.

2. Drag point A or B (or both) in order to create 3 different right triangles (ΔABC). Enter the information indicated in the table below.

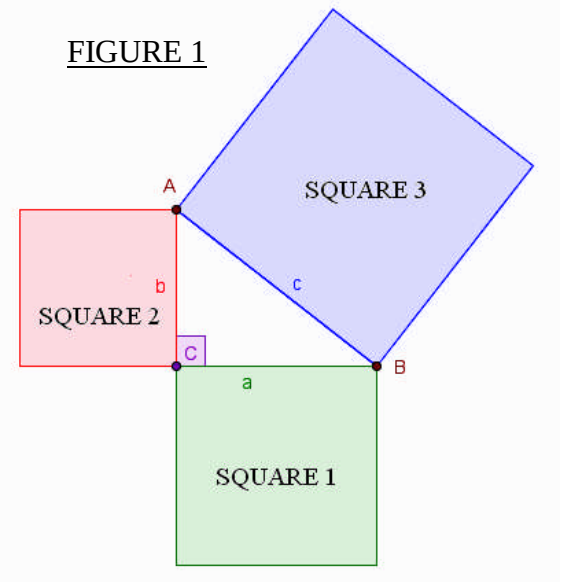
TABLE 1

	a	b	c	Area of Square 1	Area of Square 2	Area of Square 3
Triangle 1						
Triangle 2						
Triangle 3						

3. According to your findings in Table 1, how do the areas of the three squares relate to one another?

4. Refer to Figure 1 on the right.
   - **(a)** Write an expression for the area of Square 1 [Hint: The answer will be an expression in terms of a.]
   - **(b)** Write an expression for the area of Square 2 [Hint: The answer will be an expression in terms of b.]
   - **(c)** Write an expression for the area of Square 3 [Hint: The answer will be an expression in terms of c.]

FIGURE 1



5. Describe how the information you gathered here in part 1 proves the Pythagorean Theorem.

PART 2: THE LAW OF COSINES

In part 1, we used the applet to prove the validity of the Pythagorean Theorem. The Pythagorean Theorem works for right triangles, but what if the triangle is not a right triangle?

(A) ACUTE, OBTUSE, OR RIGHT

1. Drag the points A and B so that sides a and b are reasonably close to one another but not equal. (The vales of a and b should within one unit of one another.)

2. Drag the slider to create the angles indicated in column 1 of the table below. Record your findings. (When filling in the 4th column use one of the following expressions: $a^2 + b^2 > c^2$, $a^2 + b^2 = c^2$, or $a^2 + b^2 < c^2$)

a = _____ b = _____

TABLE 2

Measure of Angle C	Area of Square 1 + Area of Square 2 [$a^2 + b^2$]	Area of Square 3 [c^2]	How Do Colums 1 and 2 Compare?	Type of Triangle (Acute, Obtuse, or Right)
50°				
60°				
70°				
80°				
90°				
100°				
110°				
120°				
130°				

3. Use the observations you made in Table 2 to answer the following questions.

(a) If an angle is acute then $a^2 + b^2$ will be _____ c^2.

(b) If an angle is right then $a^2 + b^2$ will be _____ c^2.

(c) If an angle is obtuse then $a^2 + b^2$ will be _____ c^2.

Now complete the exercises on the following page.

Identify the type of triangle given the measurements of the sides. Show calculations to support your answer.

1. 9, 15, 17

2. 7, 24, 25

3. 6, 14, 16

4. 11, 12, 8

5. 6, 9, 14

6. 10, 8, 6

7. 4.5, 6.66, 9.78

8. 5.12, 8.75, 12.66

(B) DEFECTS OF A TRIANGLE

1. On the applet on the webpage, there are two check boxes, "Show Defect 1" and "Show Defect 2." When these two check boxes are checked rectangular regions appear, as well as their areas. Click on these 2 check boxes and activate them.

2. You will explore the figure produced as before, changing the angle measurement of C, but now you will be focusing attention on the areas of these defects. Use your explorations of the figure now to complete Table 3 on the next page. Keep your sides the same as they were in section A.

TABLE 3

Measure of Angle C	Area of Square 1 + Area of Square 2 [$a^2 + b^2$]	Area of Defect 1	Area of Defect 2	Area of Square 1 + Area of Square 2 + [Area of Defect 1 + Area of Defect 2]	Area of Square 1 + Area of Square 2 – [Area of Defect 1 + Area of Defect 2]	Area of Square 3 [c^2]	Type of Triangle (Acute, Obtuse, or Right)
50°							
60°							
70°							
80°							
90°							
100°							
110°							
120°							
130°							

3. What do you notice about the areas of the 2 defects?

4. Which formula (from column 5 or column 6) gives approximately the same value as the area of square 3 for the acute triangles?

5. Which formula (from column 5 or column 6) gives approximately the same value as the area of square 3 for the obtuse triangles?

6. Which formula (from column 5 or column 6) gives approximately the same value as the area of square 3 for the right triangle? Explain why this happens.

(C) CALCULATING DEFECTS ALGEBRAICALLY

1. According to Figure 2, what is the expression for the area of Defect 1?

Area = _____

2. Using Figure 2, write an expression for cos(C).

cos(C) = _____

3. Solve the expression you obtained in #2 for *x*.

x = _____

4. Now substituting the expression you obtained in #3 in for the value of *x* you have obtained in #1, what is the formula for finding the area of Defect 1?

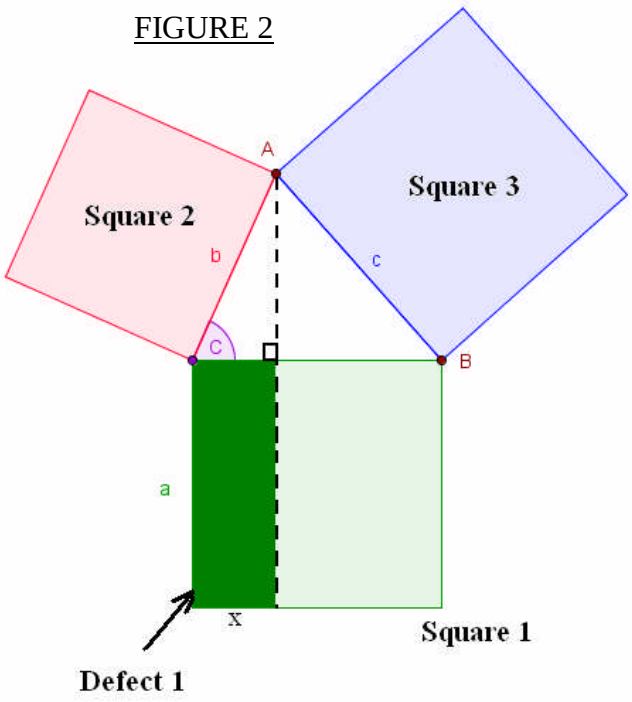
Area of Defect 1 = _____

5. Now repeat steps 1-4 above and use Figure 3 to compute the area of defect 2.

Area of Defect 2 = _____

6. Do the results from #1-5, confirm your findings in Section B #3?

FIGURE 2



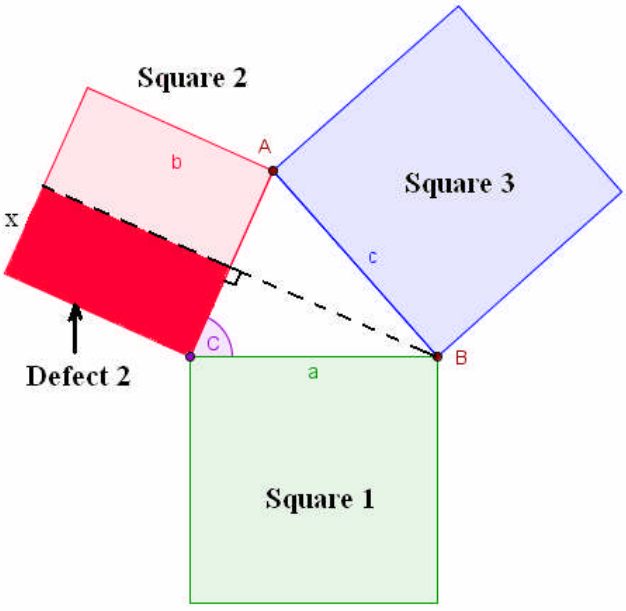


FIGURE 3

(D) PUTTING IT ALL TOGETHER

1. Assuming that the triangle is **ACUTE**, use the formula from section B #4 to complete the following.

Area of Square 3 = _____

2. Rewrite the formula above in terms of a, b, and c. You should also use the formulas you obtained from section C. [Combine like terms when possible.]

c^2 = _____

3. According to the Law of Cosines what is the value of c^2?

c^2 = _____

4. Is the value of cos(C) positive or negative when angle C is acute?

5. Is the value of cos(C) positive or negative when angle C is obtuse?

6. What is the value of cos(C) when angle C is a right angle?

7. Use the results above to describe how the Law of Cosines is still true for obtuse triangles.

8. Use any and all results from Part 2 to explain how the Law of Cosines is still true for a right triangle.

PART 3: THE LAW OF SINES

In order to complete Part 2, you will need to use the applet found on the webpage entitled BASIC TRIANGLE which can be found by following the link below:

http://atsorren.freewebpages.org/GENERAL/BASTRIANGLE/triangle.html

1. Move points A, B, and C to create 6 different triangles. Make sure to have both acute and obtuse triangles. Enter the information in Table 4 below. [Round your answers for the last three columns to 3 decimal places.]

TABLE 4

	a	b	c	$m\angle A$	$m\angle B$	$m\angle C$	$\frac{\sin(A)}{a}$	$\frac{\sin(B)}{b}$	$\frac{\sin(C)}{c}$
Triangle 1									
Triangle 2									
Triangle 3									
Triangle 4									
Triangle 5									
Triangle 6									

2. Are the values found in the last three columns equal to one another (ignore rounding errors)?

3. According to your book and notes, what is the Law of Sines?

4. Did your results in Part 3 confirm the validity of the Law of Sines? Explain.