

PRECALCULUS 11

2008-2009 FINAL REVIEW QUESTIONS

FUNCTIONS/EQUATIONS/INEQUALITIES

1. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{3x-5}$, find the following. Where appropriate include domain restrictions.

(A) $(f + g)(2)$

(B) $(g \cdot f)(2)$

(C) $(f \circ g)(2)$

(D) $(g \circ f)(2)$

(E) $\left(\frac{f}{g}\right)(x)$

(F) $\left(\frac{g}{f}\right)(x)$

(G) $(g - f)(x)$

(H) $(3f + 4g)(x)$

2. Find the inverse of the following functions.

(A) $f(x) = \sqrt{4x+1}$

(B) $f(x) = \frac{3x+5}{4}$

(C) $f(x) = \frac{6x+5}{2x-3}$

(D) $f(x) = 5\log_6(x-4) + 1$

3. Solve the following equations.

(A) $16x + 14 = 25$

(B) $8x^2 - 6x + 5 = 4$

(C) $|2x + 6| = 10$

(D) $|4x - 3| = x + 3$

4. Solve the following inequalities. Show all work.

(A) $|x + 7| > 9$

(B) $x^2 - 7x + 11 < 1$

(C) $x^2 - 4x + 8 \geq x - 3$

(D) $|2x + 7| \leq 3x - 5$

(E) $|x^2 - 6x + 5| > 2x + 1$

5. (A) Sketch a graph of the function $f(x) = \begin{cases} 2x+3 & \text{if } -4 \leq x \leq -1 \\ x^2 - 2 & \text{if } -1 < x \leq 4 \end{cases}$

Now using the graph above as a parent function...

(B) Sketch a graph of $y = -f\left(\frac{1}{2}x+1\right)+3$

(C) Sketch a graph of $y = |f(x)|$

(D) Sketch a graph of $y = f(|x|)$

EXPONENTIAL & LOGARITHMIC FUNCTIONS

6. Solve the following equations.

(A) $3^{2x+7} = 8$

(B) $3\log_6(x-4)+1 = 7$

(C) $6^{x-3} = 4^{2x+1}$

(D) $\log_4(x+1) + \log_4(x-2) = \log_4(2x+4)$

(E) $4e^{3x-1} + 7 = 15$

(F) $\ln(2x-7) + \ln(x+4) = 0$

(G) $5 \cdot 2^{4x+5} + 7 = 2$

(H) $\log_8(6x+4) = 3 - \log_8(x-2)$

7. If you put money into an account with 8.4% A.P.R. compounded quarterly, how long will it take for the value of the account to double provided that you are not depositing any additional money after the first deposit?

8. The amount of ozone in the atmosphere has been decaying at a rate of 1.59% every 3 years. Assuming that these atmospheric conditions continue...

(A) What proportion of the current amount of ozone will remain 20 years from now?

(B) What is the half-life of ozone under the current atmospheric conditions?

LIMITS & DERIVATIVES

9. Find the values of the following limits.

$$(A) \lim_{x \rightarrow 5} \frac{3x^2 - 12}{x + 2}$$

$$(B) \lim_{x \rightarrow -2} \frac{3x^2 - 12}{x + 2}$$

$$(C) \lim_{x \rightarrow 10} \frac{x^2 - 6x - 40}{x^2 - 14x + 40}$$

$$(D) \lim_{x \rightarrow 4} \frac{x^2 - 6x - 40}{x^2 - 14x + 40}$$

$$(E) \lim_{x \rightarrow -7} \frac{\sqrt{x+7}}{x+7}$$

$$(F) \lim_{x \rightarrow 2\pi} \sin\left(\frac{x}{6}\right)$$

10. Use the definition of a derivative to find the derivatives of the following functions.

$$(A) f(x) = 4 - 12x$$

$$(B) f(x) = 8x^2$$

$$(C) f(x) = \sqrt{6x+7}$$

$$(D) f(x) = 2x^3 + 3x - 4$$

POLYNOMIAL FUNCTIONS

11. For each of the functions listed below, do the following...

- (i) Find the x -intercepts [depending on the situation, this may require Newton's Method.]
- (ii) Find the y -intercept.
- (iii) Find the coordinates of all critical points and describe where the function is increasing and where it is decreasing.
- (iv) Find the coordinates of all inflection points and describe where the function is concave up and where it is concave down.
- (v) Sketch a graph of the function.

$$(A) f(x) = x^3 - 2x^2 - 9x + 18$$

$$(B) f(x) = \frac{1}{5}x^5 - \frac{8}{3}x^3 + 15x + 11$$

$$(C) f(x) = x^4 - 10x^3 + 36x^2 - 1080x$$

12. Find ALL EXACT solutions (real and imaginary) for the following equations.

(A) $16x^3 + 35x^2 + 144x + 330 = 15$

(B) $2x^5 + 8x^4 + 12x^3 + 96x^2 + 54x + 500 = 68$

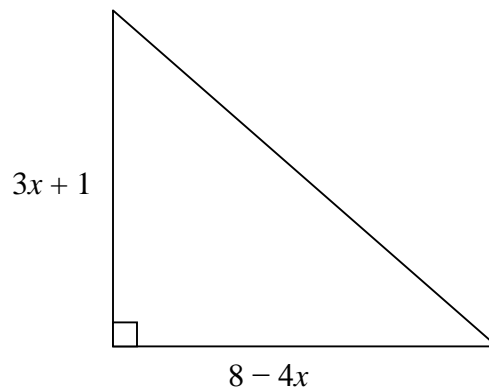
(C) $33x^6 + 816x^3 + 2720x = 272x^5 + 35x^4 + 522x^2 + 640$

OPTIMIZATION

13. You are going to construct a complete box that has a volume of 1800 cm^3 and has a rectangular base with the length equaling 1.5 times the width. What is the minimum amount of cardboard needed to construct such a box?

14. You wish to construct an open-top box by cutting away square corners from a $10 \text{ in} \times 12 \text{ in}$ piece of cardboard and folding up the sides. What is the maximum volume of such a box?

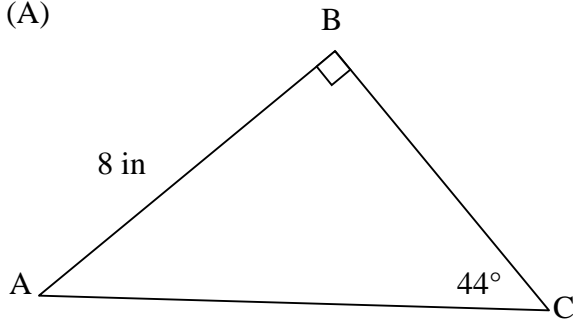
15. Given the diagram below, what value of x would produce a triangle with the maximum possible area?



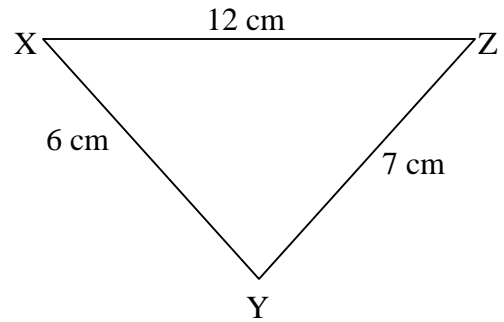
TRIGONOMETRY

16. Solve the following triangles.

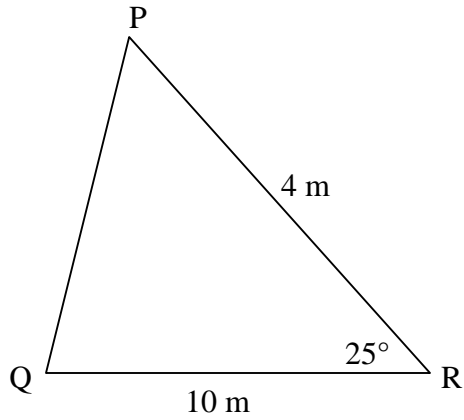
(A)



(B)



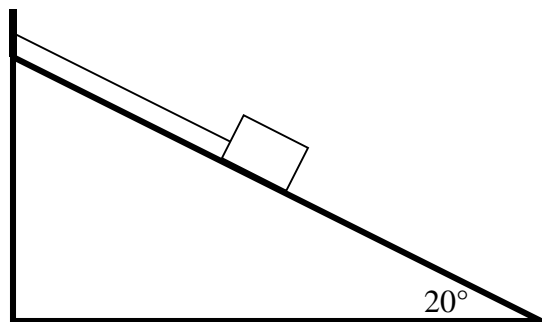
(C)



17. Find the areas of each of the triangles given in #16.

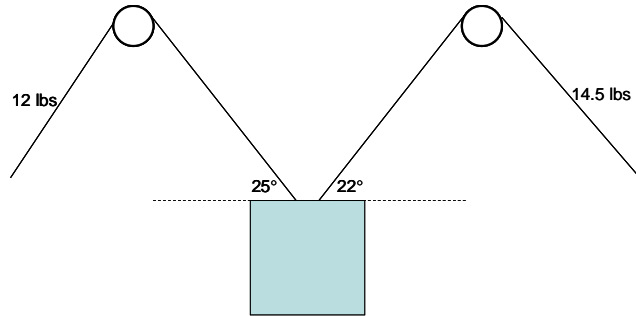
18. Given $\sin t = -\frac{3}{7}$ and $\frac{3\pi}{2} \leq t \leq 2\pi$, calculate $\cot t$

19. Given $\cos \theta = -\frac{4}{5}$ with $\frac{\pi}{2} \leq \theta \leq \pi$ and $\tan \beta = \frac{5}{12}$ with $\pi \leq \beta \leq \frac{3\pi}{2}$, find the value of the expression: $\sin \theta + \sec \beta$
20. Two people are watching a movie screen in a theater. One person is 40 ft from the screen while the other is 36 ft from the screen. The angle formed by following the path from one of the people to the movie screen and then to the other person is 31° . How far apart are the two people from each other?
21. Given $\mathbf{a} = \langle -7, 4 \rangle$ and $\mathbf{b} = \langle 3, -6 \rangle$, find the following.
- (A) $3\mathbf{a} - 4\mathbf{b}$ (B) $\|\mathbf{a}\|$ (C) The angle between \mathbf{a} and \mathbf{b} .
22. An airplane can fly at a speed of 450 mph in still air. It heads W 40° S. A wind is blowing at 15 mph in the direction S 20° E. What is the course and ground speed of the airplane?
23. What is the minimum amount of force needed to push a 200-lb object up a ramp with an incline of 13° (ignoring friction)?
24. A 100-lb weight sits motionless on a ramp with a 20-degree incline. A rope tied to the object and to a scale at the top of the ramp. Assume that the rope is running parallel to the surface of the ramp. [See the figure below.]



- (A) What is the force due to gravity acted on the object down the surface of the ramp?
- (B) The scale reads 22.5 lbs. Based on this and the results in (A) what is the force of friction between the object and the ramp?

25. An object hangs from 2 ropes attached to opposite walls through a pulley system [see figure below]. One of the ropes forms a 25-degree angle with the object and the horizontal. A scale attached to this rope shows that there is 12 lbs of tension in the rope. The second rope forms a 22-degree angle with the object and the horizontal and the tension in that rope is 14.5 lbs.



\ Based on this information, how much does the object weigh?

SEQUENCES AND SERIES\

26. For each of the following sequences: (i) Find the 20th term
(ii) Find the sum of the first 20 terms.

(A) 4, 7, 10, 13, ...

(B) 16, 8, 4, 2, ...

(C) Positive multiples of 3.

(D) Powers of 3 greater than 1.

27. Evaluate the following.

(A) $\sum_{n=1}^{25} 4n + 7$

(B) $\sum_{n=1}^{15} 3 \cdot 2^n$

(C) $\sum_{n=4}^{16} 4 \cdot 3^{n-2}$

(D) $\sum_{n=3}^{50} 5(n+4)$

(E) $\sum_{n=1}^{\infty} 1.5 \cdot 4^n$

(F) $\sum_{n=2}^{\infty} 6 \cdot \left(\frac{1}{2}\right)^{n-1}$